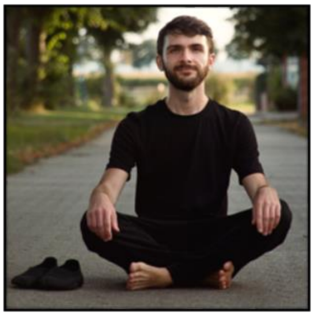
EQUATIONS OF PHYSICS



Hello! My name is Alexander Fufa**eV**. You probably know me from YouTube or my physics website fufaev.org. I am a theoretical physicist and I am **training the next Albert Einsteins and Richard Feynmans**.

In this premium Physics Formula Collection, you will find various formulas from the field of physics that do **not require knowledge of calculus**. The formula collection is therefore suitable for everyone who can get by in their education without higher mathematics, such as **all classes up to high school** and **students who have to take experimental physics**: physicists, biologists, medical students, life science students, geoscientists, meteorologists, and science education students. The lovingly designed formula collection is also a great **offline tool** that will help you solve a physics problem even without internet access.

The meaning of the formula symbols is briefly explained and the colorful formulas are visualized with illustrations.

- If a formula symbol is not explained in a formula, it is a constant whose value and explanation can be found in the chapter on **Physical constants**.
- The chapter **Alternative units** will help you convert units.
- And with the chapter **Keywords of physics** at the end of the book, you can quickly find a physical quantity.

The formula collection is also available online at en.fufaev.org/formulas. There, you can find not only the most up-to-date version of the formula, but also related content (exercises, lessons, videos) and the ability to rearrange the formula.

May the physics be with you!

A. FufaeV

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PHYSICAL CONSTANTS

Speed of light	c	$2.997\,924\,58 \cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$	exact
Elementary charge	e	$1.602\,176\,634 \cdot 10^{-19}\ \mathrm{C}$	exact
Vacuum permeability	μ_{B}	$1.256\,637\,062\,12 \cdot 10^{-6}\ \frac{\mathrm{Vs}}{\mathrm{Am}}$	
Vacuum permittivity	ε_0	$8.854\,187\,812\,8 \cdot 10^{-12}\ \frac{\mathrm{As}}{\mathrm{Vm}}$	
Planck constant	h	$6.626\,070\,15 \cdot 10^{-34}\ \mathrm{Js}$	exact
Reduced Planck constant	$\hbar$	$1.054\,571\,817\ldots \cdot 10^{-34}\ \mathrm{Js}$	
Gravitational constant	G	$6.674\,30 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$	
Boltzmann constant	k_{B}	$1.380\,649 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$	exact
Electron mass	m_{e}	$9.109\,383\,701\,5 \cdot 10^{-31}\ \mathrm{kg}$	
Proton mass	m_{p}	$1.672\,621\,923\,69 \cdot 10^{-27}\ \mathrm{kg}$	
Neutron mass	m_{n}	$1.674\,927\,498\,04 \cdot 10^{-27}\ \mathrm{kg}$	
Avogadro constant	N_{A}	$6.022\,140\,76 \cdot 10^{23}\ \frac{1}{\mathrm{mol}}$	exact
Gas constant	R	$8.314\,462\,618\,153\,24\ \frac{\mathrm{J}}{\mathrm{K\,mol}}$	exact
Atomic mass unit	μ	$1.660\,539\,066\,60 \cdot 10^{-27}\ \mathrm{kg}$	
Faraday constant	F	$9.648\,533\,212\,331\,001\,84 \cdot 10^{4}\ \mathrm{C/mol}$	exact

ALTERNATIVE UNITS

Force	F	$\mathrm{N} = \frac{\mathrm{kg\,m}}{\mathrm{s}^2}$	Newton
Angular momentum	L	$\mathrm{Js} = \mathrm{Nms} = \frac{\mathrm{kg\,m}}{\mathrm{s}}$	Joule-second
Torque	M	$\mathrm{Nm} = \frac{\mathrm{kg\,m}^2}{\mathrm{s}^2}$	Newton-meter
Angular frequency	ω	$\frac{\mathrm{rad}}{\mathrm{s}} = \frac{1}{\mathrm{s}}$	Radiant per Second
Angular acceleration	α	$\frac{\mathrm{rad}}{\mathrm{s}^2} = \frac{1}{\mathrm{s}^2}$	Radian per square second
Energy, work	W	$\mathrm{J} = \mathrm{Nm} = \frac{\mathrm{kg\,m}^2}{\mathrm{s}^2}$	Joule
Power	P	$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{kg\,m}^2}{\mathrm{s}}$	Watt
Electric charge	Q	$\mathrm{C} = \mathrm{As}$	Coulomb
Voltage	U	$\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{Nm}}{\mathrm{As}} = \frac{\mathrm{kg\,m}^3}{\mathrm{A\,s}^3}$	Volt
Electrical resistance	R	$\Omega = \frac{\mathrm{V}}{\mathrm{A}} = \frac{\mathrm{kg\,m}^3}{\mathrm{A}^2\,\mathrm{s}^3}$	Ohm
Electric field	E	$\frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{N}}{\mathrm{As}} = \frac{\mathrm{kg\,m}}{\mathrm{A\,s}^3}$	Volt per Meter
Magnetic flux density	B	$\mathrm{T} = \frac{\mathrm{Vs}}{\mathrm{m}^2} = \frac{\mathrm{kg\,m}}{\mathrm{A\,s}^2}$	Tesla
Inductance	L	$\mathrm{H} = \frac{\mathrm{Vs}}{\mathrm{A}} = \frac{\mathrm{kg\,m}^3}{\mathrm{A}^2\,\mathrm{s}^2}$	Henry
Electric capacitance	C	$\mathrm{F} = \frac{\mathrm{As}}{\mathrm{V}} = \frac{\mathrm{A}^2\,\mathrm{s}^4}{\mathrm{kg\,m}^3}$	Farad
Heat capacity	C	$\frac{\mathrm{J}}{\mathrm{K}} = \frac{\mathrm{kg\,m}^2}{\mathrm{s}^2\,\mathrm{K}}$	Joule per Kelvin
Pressure	Π	$\mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2} = \frac{\mathrm{kg}}{\mathrm{m\,s}^2}$	Pascal

UNIT PREFIXES

Prefix	Abbreviation	Decimal power
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga 	G	10^{9}
Mega 	M	10^{6}
Kilo	k	10^{3}
Deci	d	10^{-1}
Centi 	c	10^{-2}
Milli	m	10^{-3}
Micro	µ	10^{-6}
Nano 	n	10^{-9}
Pico 	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}

PHYSICAL ALPHABET

Letter	Usage	Letter	Usage	Letter	Usage
A	Area, Activity	a	Acceleration	Ξ Xi	Cascade particle
B	Magnetic flux density	b	Damping constant	Γ Gamma	Decay width
C	Capacitance	c	Specific capacitance	Π Pi	Pressure
D	Electric flux density	d	Thickness	Ω Omega	Unit: Resistance
E	Electric field	e	Elementary charge	Θ Theta	Twisting angle
F	Force	f	Frequency, focal length	Φ Phi	Magnetic flux
G	Gravitational constant	g	Gravitational acceleration	Ψ Psi	Wave function
H	Magnetic field	h	Planck constant, height	Σ Sigma	Sum sign
I	Current	i	Imaginary unit	β beta	Angle
J	Unit: Joule	j	Current density	δ delta	Loss angle
K	Unit: Kelvin	k	Boltzmann constant	∂ del	Partial derivative
L	Angular momentum, Inductance	l	Length	ϵ, ε epsilon	Vacuum permittivity
M	Torque, Mass	m	Mass	ϕ, φ phi	Phase angle
N	Particle number	n	Charge carrier density, quantum number	α alpha	Angular acceleration, angle
P	Power	p	Momentum	β beta	Angle
Q	Charge	q	Test charge	γ gamma	Weight, Lorentz factor
∇ Nabla	Derivative operator	r	Radius	$\hbar$ h quer	Reduced Planck constant
Δ Delta	Difference	ϱ rho	Mass density	η eta	Viscosity
R	Resistance	s	Distance	π pi	Pi number, π myon
S	Entropy	t	Time	σ sigma	Conductivity
T	Temperature	u	Atomic mass unit	θ, ϑ theta	Angle

Letter	Usage	Letter	Usage	Letter	Usage
U	Voltage	v	Velocity	τ tau	Half-life
V	Volume	w	Energy density	μ mü	Dipole moment
W	Energy, work	x	Space coordinate	ω omega	Angular velocity
X	Complex part of impedance	y	Space coordinate	ψ Psi	Wave function
Y	Electrical conductance	z	Space coordinate	ξ xi	Sound deflection
Z	Atomic number, impedance	λ lambda	Wavelength	Λ Lambda	Cosmological constant

CONVERSION OF UNITS

Energy conversion			
Kilowatt hour	1 kWh $\{W_{\mathrm{kWh}}\} = \{W_{\mathrm{J}}\}/3\,600\,000$	3 600 000 J $\{W_{\mathrm{J}}\} = 3\,600\,000 \cdot \{W_{\mathrm{kWh}}\}$	**Joule**
Electron volt	1 eV $\{W_{\mathrm{eV}}\} = \{W_{\mathrm{J}}\}/1.602 \cdot 10^{-19}$	$1.602\,176\,634 \cdot 10^{-19}$ J $\{W_{\mathrm{J}}\} = 1.602 \cdot 10^{-19} \cdot \{W_{\mathrm{eV}}\}$	**Joule**
Calorie	1 cal $\{W_{\mathrm{cal}}\} = \{W_{\mathrm{J}}\}/4.184$	4.184 J $\{W_{\mathrm{J}}\} = 4.184 \cdot \{W_{\mathrm{cal}}\}$	**Joule**
Pressure conversion			
Bar	1 bar $\{\Pi_{\mathrm{bar}}\} = \{\Pi_{\mathrm{Pa}}\}/100\,000$	100 000 Pa $\{\Pi_{\mathrm{Pa}}\} = 100\,000 \cdot \{\Pi_{\mathrm{bar}}\}$	**Pascal**
Millimetre of mercury	1 mmHg $\{\Pi_{\mathrm{mmHg}}\} = \{\Pi_{\mathrm{Pa}}\}/133.322$	133.322 Pa $\{\Pi_{\mathrm{Pa}}\} = 133.322 \cdot \{\Pi_{\mathrm{mmHg}}\}$	**Pascal**
Millimeter of water	1 $\mathrm{mmH_2O}$ $\{\Pi_{\mathrm{mmH_2O}}\} = \{\Pi_{\mathrm{Pa}}\}/9.80638$	9.80638 Pa $\{\Pi_{\mathrm{Pa}}\} = 9.80638 \cdot \{\Pi_{\mathrm{mmH_2O}}\}$	**Pascal**
Standard atmosphere	1 atm $\{\Pi_{\mathrm{atm}}\} = \{\Pi_{\mathrm{Pa}}\}/101\,325$	101 325 Pa $\{\Pi_{\mathrm{Pa}}\} = 101\,325 \cdot \{\Pi_{\mathrm{atm}}\}$	**Pascal**
Temperature conversion			
Kelvin	0 K $\{T_{\mathrm{K}}\} = (\{T_{\mathrm{F}}\} + \{459.67\})/1.8$	−459.67 F $\{T_{\mathrm{F}}\} = 1.8 \cdot \{T_{\mathrm{K}}\} - \{459.67\}$	**Fahrenheit**
Kelvin	0 K $\{T_{\mathrm{K}}\} = \{T_{\mathrm{C}}\} + 273.15\}$	−273.15 °C $\{T_{\mathrm{C}}\} = \{T_{\mathrm{K}}\} - \{273.15\}$	**Celsius**
Volume conversion			
Liter	1 l $\{V_{\mathrm{l}}\} = 1000 \cdot \{V_{\mathrm{m^3}}\}$	0.001 $\mathrm{m^3}$ $\{V_{\mathrm{m^3}}\} = 0.001 \cdot \{V_{\mathrm{l}}\}$	**Cubic meter**
Angle conversion			

Degree measure	1° $\{\varphi\} = \{x\} \cdot 180^\circ/\pi$	$0.01745\ \mathrm{rad}$ $\{x\} = \{\varphi\} \cdot \pi/180^\circ$	Radian measure

1. MECHANICS

About the motion of bodies and the forces acting on them.

1.1 Unaccelerated motion

$$x(t) = vt + x_0$$

The **current position** $x(t)$ [m] of a body at **time** t [s] moving with **constant velocity** v [m/s] measured from the **initial position** x_0 [m]. The body is at the **initial position** $x(0) = x_0$ at time $t = 0$.

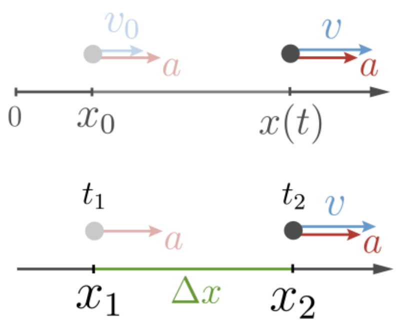
$$\Delta x = v\Delta t$$

The **distance traveled** $\Delta x = x_2 - x_1$ [m] from an arbitrary **position** $x(t_1) = x_1$ to an arbitrary **position** $x(t_2) = x_2$ is covered by the body within the **time** $\Delta t = t_2 - t_1$ [s]. If the distance traveled is measured from the start position, then $t_1 = 0$ and the distance can be calculated with $\Delta x = vt$.

1.2 Uniformly accelerated motion

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

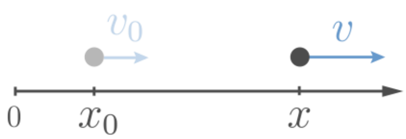
$$\Delta x = \frac{1}{2}a(t_2{}^2 - t_1{}^2) + v_0(t_2 - t_1)$$



The **current position** $x(t)$ [m] of a body at **time** t [s]. Here, the body accelerates uniformly with the constant **acceleration** a [m/s^2] and has the **start velocity** v_0 [m/s] and the **initial position** $x(0) = x_0$ [m] at time $t = 0$. Usually, the start position is set to zero: $x_0 = 0$. The covered **distance** $\Delta x = x_2 - x_1$ [m] from an arbitrary **position** $x(t_1) = x_1$ to an arbitrary **position** $x(t_2) = x_2$ is covered by the body within the **time** $\Delta t = t_2 - t_1$ [s].

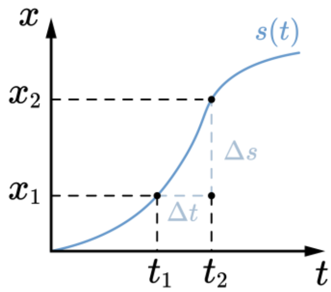
$$v(t) = v_0 + at$$

$$v(x) = \sqrt{v_0{}^2 + 2a(x - x_0)}$$



Velocity $v(t)$ after the body has experienced **acceleration** a during the **time** t and had an **initial velocity** v_0. And $v(x)$ is the velocity after the body has covered the **distance** $x - x_0$ during acceleration.

$$\bar{v} = \frac{\Delta x}{\Delta t} \qquad \bar{a} = \frac{\Delta v}{\Delta t}$$

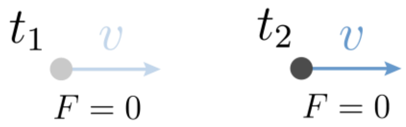


Average velocity $\bar{v}$ [m/s] of a body which travelled the **distance** $\Delta x = x_2 - x_1$ within the **time** $\Delta t = t_2 - t_1$. **Average acceleration** $\bar{a}$ [m/s^2] of a body whose **velocity** has changed by $\Delta v = v_2 - v_1$ within the **time** $\Delta t = t_2 - t_1$.

1.3 Newton's axioms

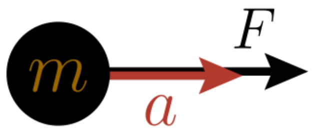
Newton's First Law of Motion

A body remains at rest ($v = 0$) or continues to move at **constant velocity** v if no forces act on the body.



Newton's Second Law of Motion

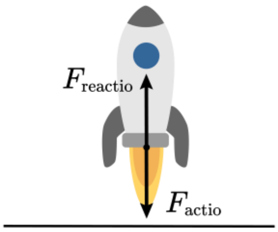
$$F = ma \qquad F = m\frac{\Delta v}{\Delta t} \qquad F = \frac{\Delta p}{\Delta t}$$



If a **resulting force** F [N] acts on a body, then the body of **mass** m [kg] experiences an **acceleration** a [m/s^2]. In other words, a change in **velocity** (magnitude or direction) $\Delta v = v_2 - v_1$ [m/s] within **time** $\Delta t = t_2 - t_1$ [s] results in a force on the body. In some cases, the mass of a moving body may also change over time (e.g., in a rocket). In this case, the **momentum change** $\Delta p = p_2 - p_1$ [kg m/s] should be considered to calculate the resulting force. The formula in the middle exploits the **N**ewton's **F**irst **L**aw of **M**otion.

Newton's Third Law of Motion

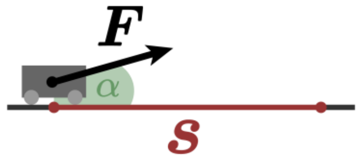
$$F_{\text{actio}} = -F_{\text{reactio}}$$



If body A exerts a **force** F_{actio} (action) on a body B, then B exerts an equally large, **oppositely directed force** F_{reactio} (reaction) on body A. Important: Interaction forces F_{actio} and F_{reactio} always act on two *different* bodies!

1.4 Work and power

$$\varDelta W = F s \cos(\alpha)$$



Work ΔW [J] is the energy gained or lost by a body (e.g. a trolley) when a force has been applied TO or BY this body along the **distance** s [m], at the **angle** α [rad].

$$\varDelta W = \varDelta W_{\mathrm{kin}} = W_{\mathrm{kin2}} - W_{\mathrm{kin1}}$$

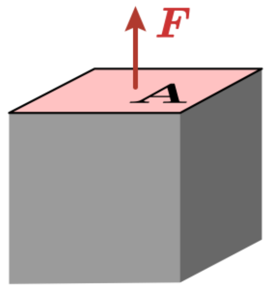
The **work** $\varDelta W$ is the difference of the **kinetic energy** ΔW_{kin} [J]. Here W_{kin1} [J] is the kinetic energy of the body before the action of the force and W_{kin2} [J] is the kinetic energy *after* the action of the force.

$$P = \frac{\Delta W}{\Delta t}$$

The (mechanical) **power** P [W = J/s] is **work** ΔW [J] per **time** t [s].

1.5 Coefficient of elasticity, mechanical strain and stress

$$\epsilon = \frac{\varDelta l}{l_0} \qquad \sigma = \frac{F}{A} \qquad E = \frac{\epsilon}{\sigma}$$

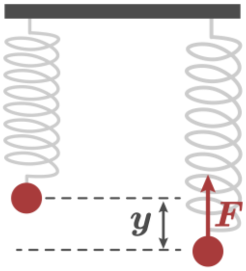


- **Mechanical strain** ϵ [−] is defined as **change in length** $\varDelta l = \mathrm{l}_1 - l_1$ [m] due to action of a force and l_0 [m] is the **initial length** (without any action of the forces).
- The **mechanical stress** σ [N/m^2] is defined as **force** F [N] per **cross-sectional area** A [m^2].
- The **coefficient of elasticity** (Young's modulus) E [N/m^2] is defined as mechanical strain ϵ divided by mechanical stress σ.

Material at 20 °C	Coefficient of elasticity E in 10^9 N/m^2
Ebonite	5
Concrete	20 to 40
Glas	40 to 90
Aluminium (Al)	70
Gold (Au)	78
Structural steel	210
Diamond, graphene	1000

1.6 Spring (harmonic oscillator) and Hooke's law

$$F = -Dy \qquad \sigma = E\epsilon$$



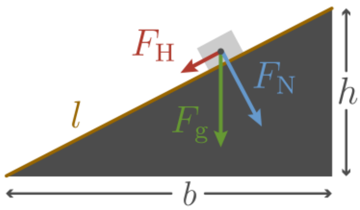
A mass attached to the spring is deflected by the **distance** y [m] and experiences a **spring force** F [N] which is directed against the deflection. The greater the **spring constant** D [N/m], which describes the stiffness of the spring, the greater the spring force. Hooke's law describes the linearity between the force F and the deflection y. Hooke's law can also be expressed by using **mechanical strain** ϵ [$-$] and **mechanical stress** σ [N/m^2], where the **coefficient of elasticity** E [N/m^2] is the constant of proportionality.

$$W_{\text{pot}} = \frac{1}{2}Dy^2$$

The **stress energy** W_{pot} [J] depends on the **deflection** y and on the **spring constant** D [N/m].

1.7 Inclined plane

$$F_{\text{H}} = \frac{h}{l}F_{\text{g}} \qquad F_{\text{N}} = \frac{b}{l}F_g$$



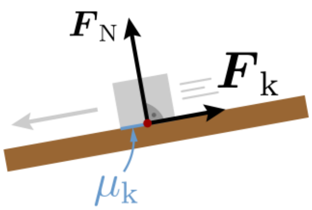
- **Slope down force** F_{H} [N] is the force on a body acting parallel to the inclined plane. It results due to the **weight force** F_{g} [N] acting on the body towards the ground. The inclined plane has **length** l [m] and **height** h [m].
- **Normal force** F_{N} [N] is the force component of the weight force acting on the body perpendicular to the inclined plane. Here, b [m] is the **width** of the support of the inclined plane - that is the length of the cathetus.

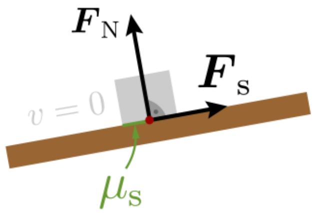
1.8 Static, kinetic and rolling friction

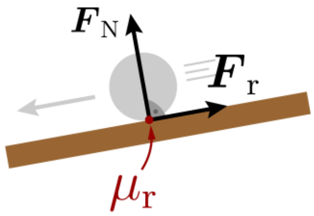
$$F_s = \mu_s F_N$$

$$F_k = \mu_k F_N$$

$$F_r = \mu_r F_N$$





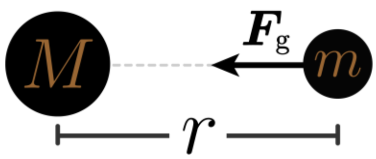


Static friction force F_s [N], **kinetic friction force** F_k [N] and **rolling friction force** F_r [N] depend on the **normal force** F_N [N], which presses the body perpendicularly onto a supporting surface, and depend respectively on the **static friction coefficient** μ_s [–], **kinetic friction coefficient** μ_k [–] or **rolling friction coefficient** μ_r [–].

Surfaces	μ_s	Surfaces	μ_k	Surfaces	μ_r
Steel on steel	0.2	Steel on steel	0.1	Tire on asphalt	0.011 to 0.015
Wood on wood	0.5	Wood on wood	0.4	Railroad wheel on rail	0.001 to 0.002
Stone on wood	0.9	Stone on wood	0.7	Tire on concrete	0.01 to 0.02
Stone on stone	1.0	Stone on stone	0.9	Tire on sand	0.2 to 0.4

1.9 Newton's law of gravity and gravitational acceleration

$$F_g = GM\frac{m}{r^2} \qquad g = GM\frac{1}{r^2}$$



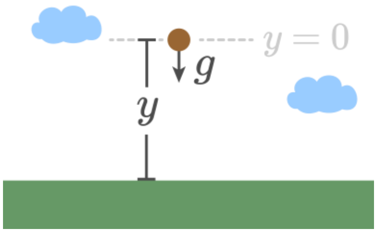
Two bodies of **masses** M [kg] and m [kg] are at a distance r [m] from each other and experience equal attracting **gravitational force** F_g [N]. Gravitational force divided by mass m gives **gravitational acceleration** g [m/s^2].

Celestial body	Gravitational acceleration g in m/s^2
Mars	3.7
Venus	8.9
Earth	9.8
Jupiter	24.8

Celestial body	Gravitational acceleration g in m/s^2
Sun	274

Table 1: Gravitational acceleration not far above the surface of the celestial body. For the distance of the celestial body to the test mass, the radius of the respective celestial body was used.

$$F_g = mg$$

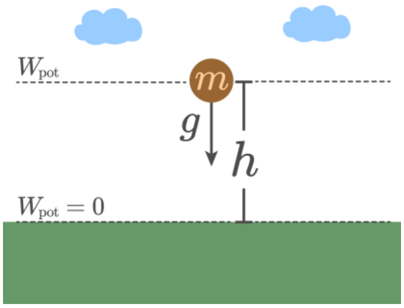


A body of **mass** m [kg] near another (large) mass, experiences a **gravitational acceleration** g [m/s^2], which leads to a **gravitational force** (approximation: **weight**) F_g

$$W_{\text{pot}} = -GM\frac{m}{r} \qquad V = -GM\frac{1}{r}$$

A **mass** m located at a **distance** r from the **mass** M has a **potential energy** W_{pot} [J]. The potential energy is negative (has a minus sign) so that the mass m has a smaller ("more negative") potential energy when it is closer to the mass M. If you are only interested in the magnitude of the potential energy, then you can omit the minus sign. Here V [J/kg] is the **gravitational potential**.

$$W_{\text{pot}} \approx mgh$$



If a body of **mass** m is not far from the earth's surface, it has a potential energy W_{pot} [J] proportional to its **height** h [m] **above the ground**. Here g [m/s^2] is the **gravitational acceleration**.

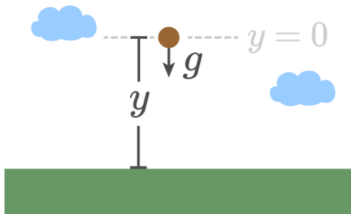
1.10 Free fall

$$y(t) = -\frac{1}{2}gt^2 + y_0 \qquad v(y) = \sqrt{2g(y_0 - y)}$$

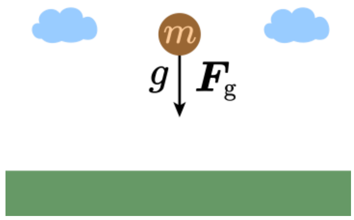
$$\Delta y = \frac{1}{2}g({t_2}^2 - {t_1}^2) \qquad t_f = \sqrt{\frac{2y_0}{g}}$$

$$v(t) = -gt$$

Position $y(t)$ [m] (above ground) of a vertically falling body at **time** t [s]. The body is dropped from the **initial height** $y(0) = y_0$ and experiences the **gravitational acceleration** $a(t) = -g$ [m/s^2]. The **distance** $\Delta y = y_2 - y_1$ [m] between height y_2 and y_1 is covered by the body within the **time** $\Delta t = t_2 - t_1$ [s]. The body lands at the **final time** t_f at the **final position** $y(t_f) = 0$ on the ground.



$$F_g = mg$$



A body of **mass** m [kg] above the earth experiences a **gravitational acceleration** g [m/s^2], which leads to a **falling force** (weight) F_g [N].

Celestial body	Gravitational acceleration g in m/s^2
Mars	3.7
Venus	8.9
Earth	9.8
Jupiter	24.8
Sun	274

1.11 Vertical throw

$$y(t) = v_0 t - \frac{g}{2} t^2 \qquad y_{max} = \frac{{v_0}^2}{2g}$$

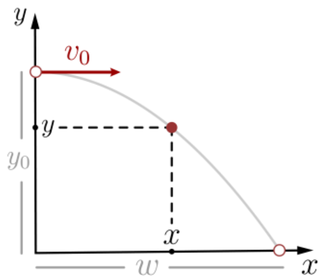
$$v(t) = v_0 - gt \qquad t_s = \frac{v_0}{g}$$

Height $y(t)$ [m] of a body thrown vertically upwards after **time** t [s]. The **instantaneous velocity** $v(t)$ [m/s] depends on the **initial velocity** v_0 [m/s] and the **gravitational acceleration** $g = 9.8$ m/s. Here t_s [s] is the **time of climb** and y_{max} [m] is the **maximum height** of throw.

1.12 Horizontal throw

$$y(x) = -\frac{g}{2{v_0}^2} x^2 + y_0 \qquad y(t) = -\frac{1}{2} g t^2$$

Height $y(x)$ [m] above the ground of a horizontally thrown body after it has traveled the **horizontal distance** x [m]. The body is thrown horizontally from the **initial height** y_0 [m] with the **initial velocity** v_0[m/s]. Of course, the body can also have been shot. Here $y(t)$ [m] is the **height** at **time** t.



$$v_y(y) = \sqrt{v_{y0} - 2gy} \qquad v_y(t) = v_{y0} - gt$$

Velocity v_y [m/s] above the ground at **height** y [m] or after **flight time** t [s] when the body is thrown with **initial velocity** v_{y0} [m/s] in horizontal x-direction.

$$v_x = v_{x0} \qquad x(t) = x_0 + v_{x0}t \qquad t_d = \sqrt{\frac{2y_0}{g}} \qquad w = v_{x0}\sqrt{\frac{2y_0}{g}}$$

Duration t_d [s] of a horizontal throw depends on the **initial height** y_0. The **flight distance** w [m] of a horizontal throw additionally depends on the **initial horizontal velocity** v_{x0} [m/s].

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

Current **total velocity** v [m/s] of a body thrown horizontally (or obliquely), at a certain time or height, is composed of the **horizontal velocity** v_x [m/s] in x-direction and the **vertical velocity** v_y [m/s] in y-direction.

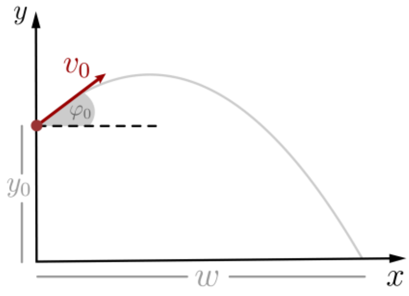
$$\varphi = \arctan\left(\frac{v_y}{v_x}\right)$$

Angle of impact φ [rad] is the angle between the horizontal x-axis and the direction of the total velocity v. The formula also applies to an oblique throw.

1.13 Oblique throw

$$y(t) = y_0 - v_{y0}t - \frac{1}{2}gt^2$$

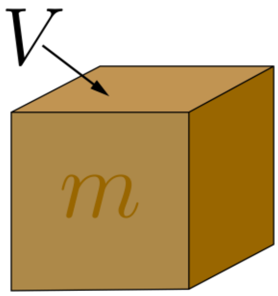
$$y(x) = -\frac{g}{2{v_0}^2\cos(\varphi_0)^2}x^2 + \tan(\varphi_0)\,x + y_0$$



Height $y(x)$ [m] above the ground of an obliquely thrown body after it has traveled the **horizontal distance** x [m]. The body is thrown from the **initial height** y_0 [m] with the **initial velocity** v_0 [m/s] at an **angle** φ_0 [rad]. Of course, the body may also have been shot. For the calculation of the **current height** $y(t)$ as a function of **time** t [s] the **initial vertical velocity** v_{y0} in y-direction is necessary.

1.14 Density and specific weight

$$\rho = \frac{m}{V}$$



Mass density ρ [kg/m³] of a solid or fluid depends on its **mass** m [kg] and its **volume** V [m³].

$$\gamma = \frac{F_g}{V} \qquad \gamma = \frac{mg}{V} \qquad \gamma = \rho g$$

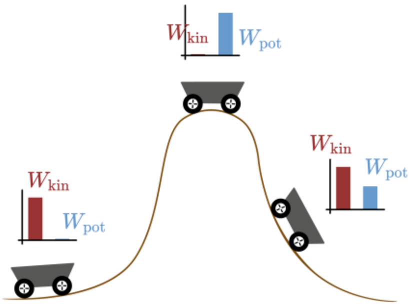
The **specific weight** γ [N/m³] of a body or fluid depends on the **gravitational force** F_g [N] exerted by the earth on the body and on the **volume** V of the body. Thus, the specific weight is the **mass density** ρ weighted by the **gravitational acceleration** g [m/s²].

Material at 20 °C		**Mass density ρ in kg/m³**	**Specific weight γ in N/m³ on the earth**
Helium	Gas	*0.18*	*1.76*
Air	Gas	*1.29*	*12.6*
Water	Liquid	*1000*	*9800*
Iron (Fe)	Liquid	*7874*	*77 165*
Gold (Au)	Solid	*19 302*	*189 159*

1.15 Conservation of energy in the gravitational field

$$W = W_{\text{kin}} + W_{\text{pot}}$$

$$W = \frac{1}{2}mv^2 + mgh$$



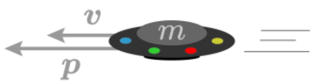
The **total energy** W [J] of a body of **mass** m is the sum of its **kinetic energy** W_{kin} [J] and its **potential energy** W_{pot} [J].

$$\frac{1}{2}mv_1^{\,2} + mgh_1 = \frac{1}{2}mv_2^{\,2} + mgh_2$$

According to the *law of conservation of energy*, the sum of the kinetic and potential energy (total energy) of the body at any time always has the same value. Here, v_1 [m/s] is the **velocity** and h_1 [m] is the **height** of a body above the ground at **time** t_1 and v_2 is the velocity and h_2 is the height at another time t_2. The **mass** m can be canceled.

1.16 Momentum and collisions

$$p = mv$$



A body of **mass** m [kg] moving with **velocity** v [m/s] has (**mechanical**) **momentum** p [kg · m/s].

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

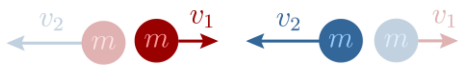
Two bodies collide *elastically*. Elastic means that the conservation of energy is fulfilled. The first body has **mass** m_1 [kg] and **velocity** v_1 [m/s] and the second body has **mass** m_2 [kg] and **velocity** v_2 [m/s] *before* the collision. *After* the collision, the masses of the bodies remain the same, but the velocities of the first and second bodies change to v'_1 [m/s] and v'_2 [m/s] (*conservation of momentum*).

$$v_2' = v_1\left(\frac{2m_1}{m_1 + m_2}\right) + v_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

$$v_1' = v_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + v_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

Two bodies collide elastically head-on (**central elastic collision**). It is assumed here that the body does not deform during the collision or rotate after the collision. The **velocity of the second body** after the collision is v_2'. The **velocity of the first body** after the collision is v_1'.

$$v_1' = v_2 \quad v_2' = v_1$$



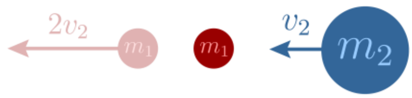
After an elastic central collision of two equal masses, the velocities of the two bodies are reversed.

$$v_2' = v_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

$$v_1' = v_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

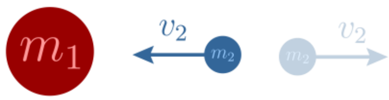
Elastic central collision of a second body with **stationary** first body.

$$v_1' = 2v_2 \quad v_2' = v_2$$



Elastic central collision of a **heavy** second body with a **resting, light** first body.

$$v_1' = 0 \qquad v_2' = -v_2$$

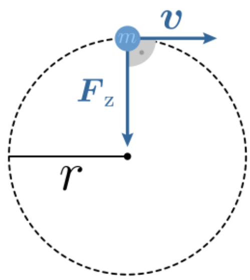


Elastic central collision of a **light** second body, with a **resting heavy** first body.

$$\Delta p = F\Delta t$$

Impulse $\Delta p = p_2 - p_1$ [Ns = kg · m/s] is (approximately) the difference between **final momentum** p_2 and **initial momentum** p_1. Thus, the impulse Δp is a change of momentum of a body due to the force F [N] applied to the body within the **time** $\Delta t = t_2 - t_1$ [s].

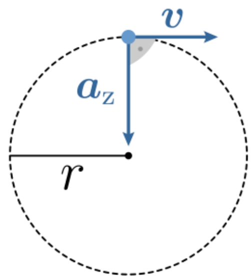
1.17 Uniform (unaccelerated) circular motion



$$F_z = \frac{mv^2}{r} \qquad F_z = m\omega^2 r$$

$$F_z = m\frac{4\pi^2 r}{T^2} \qquad F_z = 4m\pi^2 f^2 r$$

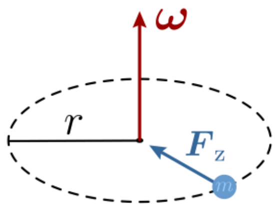
A body of **mass** m [kg] and **orbital velocity** v [m/s] is held on a circular path of **radius** r [m] by a **centripetal force** F_z [N].



The centripetal force can also be expressed with the **angular velocity** ω [rad/s]. The angular velocity is *linearly* related to the orbital velocity: $v = r\omega$. The angular frequency is related to the **frequency** f [Hz] as follows: $\omega = 2\pi f$. And the **period** T [s] of one revolution is $T = 1/f$.

$$a_z = \frac{v^2}{r} \qquad a_z = \omega^2 r$$

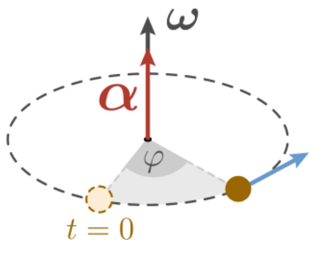
$$a_z = \frac{4\pi^2 r}{T^2} \qquad a_z = 4\pi^2 f^2 r$$



The body experiences a **centripetal acceleration** a_z [m/s^2], which points in the same direction as the centripetal force. You can calculate the centripetal acceleration either with the **orbital velocity** v [m/s], with the **angular velocity** ω [rad/s], with the **period** T [s] or with the **frequency** f [Hz].

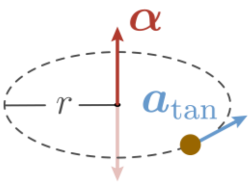
1.18 Non-uniform (accelerated) circular motion

$$\varphi(t) = \frac{1}{2}\alpha t^2 + \omega_0 t$$



A body, which experiences an **angular acceleration** α [rad/s^2] on a circular path and starts with the **initial angular velocity** ω_0 [rad/s], covers an **angle** $\varphi(t)$ [rad] within the **time** t. The angular acceleration is linearly related to the **tangential (orbital) acceleration** a_{tan} [m/s^2]: $a_{tan} = r\alpha$.

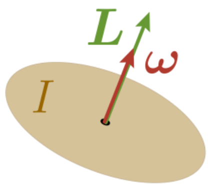
$$a = \sqrt{a_{\text{tan}}{}^2 + a_z{}^2}$$



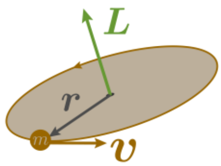
Total acceleration a [m/s^2] is composed of **tangential acceleration** a_{tan} [m/s^2] pointing along the circular path and **centripetal acceleration** a_z [m/s^2] pointing to the center of the circle.

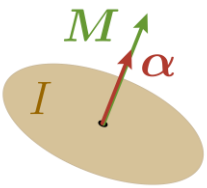
1.19 General quantities of rotation

$$L = I\omega \qquad L = mr^2\omega \qquad L = mrv$$



Angular momentum L [Js] is the product of the **moment of inertia** I [kg · m^2] of the rotating body and its **angular velocity** ω [rad/s]. A body of **mass** m [kg] performs a circular motion at a **distance** r [m] from the axis of rotation with angular velocity ω or with **orbital velocity** v [m/s].

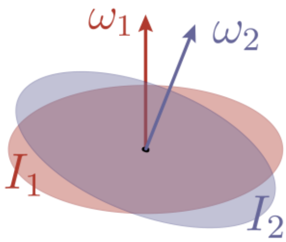




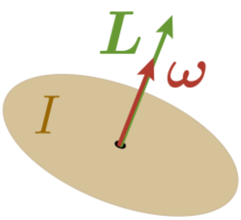
$$M = I\alpha$$

Newton's Second Law for Rotation describes how **torque** M [Nm] is related to **angular acceleration** α [rad/s^2] and **moment of inertia** I [kg · m^2].

$$I_1\omega_1 = I_2\omega_2$$



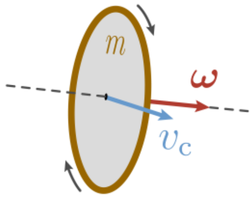
Law of conservation of angular momentum states that the product of **moment of inertia** I_1 [kg · m^2] and **angular velocity** ω_1 [rad/s] at a given time t_1 must be equal to the product of **moment of inertia** I_2 [kg · m^2] and **angular velocity** ω_2 [rad/s] at another (e.g. later) time t_2.



$$W_{\text{rot}} = \frac{1}{2}I\omega^2$$

The **rotational energy** W_{rot} [J] of a rigid body depends on its **moment of inertia** I [kg · m^2] and **angular velocity** ω [rad/s]. The axis of rotation is assumed to be fixed here.

$$W = W_{\text{kin}} + W_{\text{rot}} \qquad W = \frac{1}{2} m v_{\text{c}}^2 + \frac{1}{2} I_{\text{c}} \omega^2$$

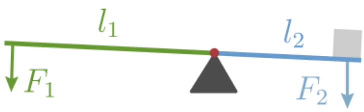


$$m v_{\text{c1}}^2 + I_{\text{c}} \omega_1^2 = m v_{\text{c2}}^2 + I_{\text{c}} \omega_2^2$$

In a rolling motion, the **total energy** W [J] of the body is composed of its **rotational energy** W_{rot} [J] and the **kinetic energy** W_{kin} [J] of the translational motion. The rotational energy depends on the **moment of inertia** I_{c} [$\text{kg} \cdot \text{m}^2$] about an axis of rotation through the center of gravity. The kinetic energy depends on the **linear velocity** v_{c} [m/s] of the center of gravity of the body and its **total mass** m [kg]. According to the *law of conservation of energy*, the sum of the kinetic energy and the rotational energy (total energy) at any time t_1 is equal to the total energy at any later time t_2.

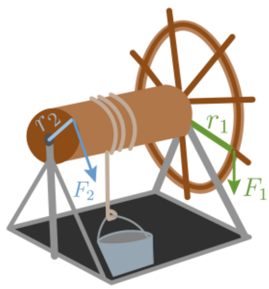
1.20 Lever law and its applications

$$F_1 l_1 = F_2 l_2 \qquad m_1 l_1 = m_2 l_2$$



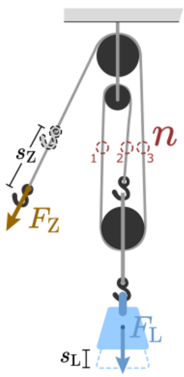
A body of **mass** m_2 [kg] lies on the load arm of **length** l_2 [m] and exerts a **gravitational force** (**weight**) F_2 [N] on the load arm. To lift this mass, a **force** F_1 [N] must be exerted on the load arm of **length** l_1 [m]. This formula is called the *Lever law*.

$$F_1 r_1 = F_2 r_2$$



A **wheel and axle** consists of two wheels of different sizes, which are connected by an axle. The Lever law also applies to such a system. The lengths here correspond to the **radii** r_1 [m] and r_2 [m] of the two wheels.

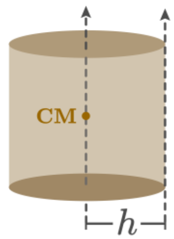
$$F_{\text{Z}} = \frac{1}{n} F_{\text{L}} \qquad s_{\text{Z}} = n s_{\text{L}} \qquad F_{\text{Z}} s_{\text{Z}} = F_{\text{L}} s_{\text{L}}$$



A body is attached to a **pulley block** with n [−] as the **number of load-bearing ropes**, which exerts a **load force** F_{L} [N] on one end of the rope. The **pulling force** F_{Z} [N] is the opposing force that lifted the body. Here, s_{Z} [m] is the **distance** by which the rope must be pulled to lift the body by the **distance** s_{L} [m].

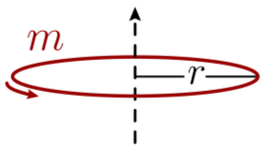
1.21 Moments of inertia of extended bodies

$$I = I_{\mathrm{CM}} + mh^2$$



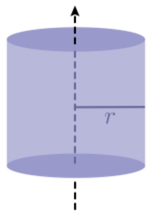
Using **Parallel Axis Theorem**, you can calculate the **moment of inertia** I [$\mathrm{kg} \cdot \mathrm{m}^2$] about an axis of rotation shifted parallel by the **distance** h [m]. The original axis of rotation passing through the center of mass CM has **moment of inertia** I_{CM} [$\mathrm{kg} \cdot \mathrm{m}^2$]. The body has **total mass** m [kg].

$$I = mr^2$$



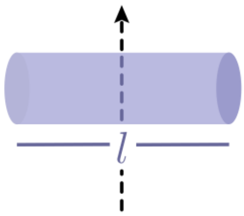
A **thin ring** with homogeneously distributed **mass** m [kg] rotates at a **distance** r [m] from the axis of rotation **passing through the center of the ring** and thus has a **moment of inertia** I [$\mathrm{kg} \cdot \mathrm{m}^2$].

$$I = \frac{1}{2}mr^2$$



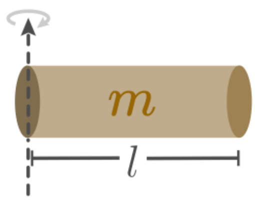
A **cylinder** with **radius** r and with homogeneously distributed **mass** m, rotates about an axis of rotation passing **through its longitudinal axis** (perpendicular to the radius).

$$I = \frac{1}{12}ml^2$$



A **rod** of **length** l with homogeneously distributed **mass** m, rotates about an axis of rotation **perpendicular to its longitudinal axis**.

$$I = \frac{1}{3}ml^2$$



A **rod** of **length** l with homogeneously distributed **mass** m, rotates about an axis of rotation passing **through one end of the rod**.

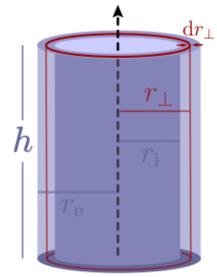
$$I = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$$

A **solid cylinder** of **length** l and **radius** r, with **mass** m uniformly distributed, rotates about an axis **perpendicular to its longitudinal axis** passing through its center.

$$I = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$$

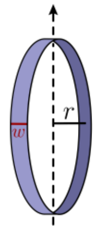
A **hollow cylinder** of **length** l and **radius** r, with **mass** m uniformly distributed, rotates about an axis **perpendicular to its longitudinal axis** passing through its center.

$$I = \frac{1}{2}m({r_e}^2 + {r_i}^2)$$



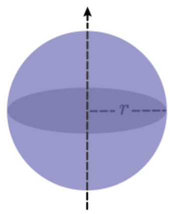
A **hollow cylinder** with an **outer radius** r_e [m] and an **inner radius** r_i [m] and a **mass** m homogeneously distributed on its surface, rotates about an axis of rotation passing **through its longitudinal axis** (perpendicular to the radius).

$$I = \frac{1}{2}m\left(r^2 + \frac{w^2}{6}\right)$$



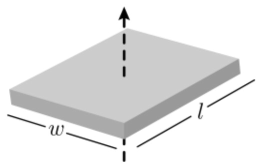
A **hollow cylinder** of **width** w [m] and **radius** r [m], whose **mass** m is homogeneously distributed on its surface, rotates about an axis of rotation **parallel to the radius through the center**.

$$I = \frac{2}{5}mr^2$$



A **sphere** with **radius** r and with homogeneously distributed **mass** m, rotates about an axis of rotation passing **through the center**.

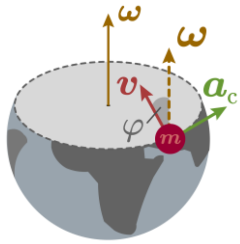
$$I = \frac{1}{12}m(l^2 + w^2)$$



A **cuboid** of **width** w [m] and **length** l [m], whose **mass** m is homogeneously distributed, rotates about an axis of rotation passing **through the center**.

1.22 Fictitious forces (Coriolis force)

$$F_{\mathrm{c}} = 2mv\omega\sin(\varphi) \qquad a_{\mathrm{c}} = 2v\omega\sin(\varphi)$$



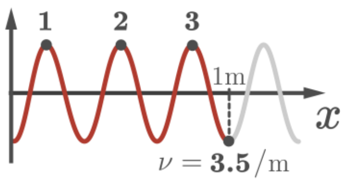
The **Coriolis force** F_{c} [N] and the **Coriolis acceleration** a_{c} [m/s²] act on a body of **mass** m [kg] flying above the planet rotating with **angular velocity** ω [rad/s] and with **velocity** v [m/s]. Here the velocity vector and the angular velocity enclose an **angle** φ [rad].

2. OSCILLATIONS AND WAVES

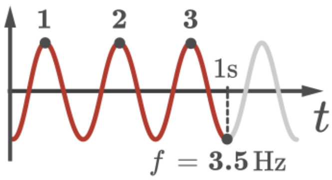
Description of light and sound waves and everything that vibrates.

2.1 Characteristic quantities of an oscillation

$$\nu = \frac{1}{\lambda} \qquad \omega = 2\pi f$$



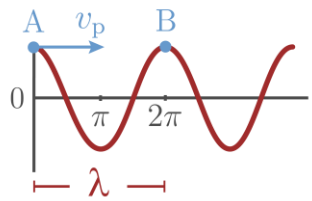
Wavenumber ν [1/m] (pronounced: Nu) is the reciprocal of the **wavelength** λ [m] and indicates the number of oscillations per meter. The **angular frequency** ω [rad/s] is related to the **frequency** f [1/s = Hz] via the factor 2π.



$$k = \frac{2\pi}{\lambda} \qquad k = 2\pi\nu$$

Angular wavenumber k [rad/m] is defined as maximum phase angle 2π per **wavelength** λ [m]. Angular wavenumber is related to (ordinary) **wavenumber** ν [1/m] by factor 2π.

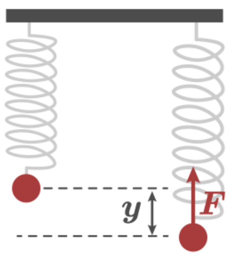
$$v_{\mathrm{p}} = \frac{f}{\nu} \qquad v_{\mathrm{p}} = \frac{\omega}{k} \qquad v_{\mathrm{p}} = \lambda f$$



The **phase velocity** v_{p} [m/s] is the ratio of the **frequency** f [Hz] to the **wavenumber** ν [1/m] or alternatively the ratio of the **angular frequency** ω [rad/s] to the **angular wavenumber** k [rad/m]. However, the phase velocity can also be written as the product of the **wavelength** λ [m] and the frequency f.

2.2 Undamped harmonic oscillation

$$y(t) = A\cos(\omega t + \varphi)$$
$$v(t) = -\omega A\sin(\omega t + \varphi)$$
$$a(t) = -\omega^2 A\cos(\omega t + \varphi)$$



Position $y(t)$ [m] at **time** t [s] of an **undamped harmonic oscillator**, such as an oscillating mass on a spring. **Velocity** $v(t)$ [m/s] at time t and **acceleration** $a(t)$ [m/s^2] at time t. The **amplitude** A [m] is then the deflection of the mass from the equilibrium position. The **angular frequency** ω [rad/s] describes how fast the mass oscillates and the **phase** φ [rad] specifies the deflection at time $t = 0$.

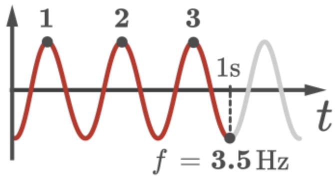
$$v_{\max} = \sqrt{\frac{D}{m}} A \quad a_{\max} = \frac{D}{m} A$$

Maximum velocity $v_{\max}$ [m/s] and **maximum acceleration** $a_{\max}$ [m/s^2] of a harmonic oscillation. Here A [m] is the **amplitude**, D [kg/s^2] the **spring constant**, m [kg] is the **mass** and $\sqrt{D/m} = \omega$ [rad/s] the **angular frequency**.

$$v(x) = \pm\sqrt{\frac{D}{m}(A^2 - x^2)}$$

Velocity $v(x)$ [m/s] of the oscillator (attached mass) as a function of its current **position** x [m].

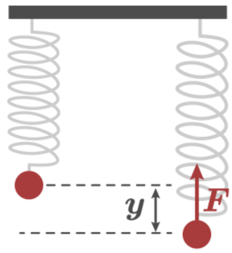
$$f = \frac{1}{2\pi}\sqrt{\frac{D}{m}} \quad T = 2\pi\sqrt{\frac{m}{D}}$$



Frequency f [Hz = 1/s] or **period** T [s] of a harmonic oscillation tells how fast, for example, a body oscillates on a spring. The frequency is independent of the deflection and depends only on the **spring constant** D [N/m] and the **mass** m [kg] of the body hanging on the spring. In the case of a spring whose mass cannot be neglected, part of the spring mass must also be taken into account in the mass m because the spring also oscillates.

2.3 Damped harmonic oscillation

$$y(t) = A e^{-(b/2m)t} \cos(\omega t + \varphi)$$



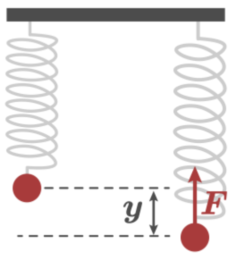
Position $y(t)$ [m] at **time** t [s] of a **linearly damped** harmonic oscillator. Here A [m] is the **amplitude** (maximum deflection), b [kg/s] is the **damping constant**, m [kg] is the **mass** of the oscillator, ω [rad/s] is the **excitation angular frequency** at which the oscillator oscillates and φ [rad] is a **phase constant** which determines the deflection $y(0)$ at time $t = 0$.

$$f = \frac{1}{2\pi}\sqrt{\frac{D}{m} - \frac{b^2}{4m^2}}$$

Frequency f $[\mathrm{Hz} = 1/\mathrm{s}]$ indicates how fast the damped harmonic oscillator oscillates. For example, how fast a mass hanging on a spring oscillates. In contrast to an undamped oscillation, the frequency of a damped oscillation is lower because of the **damping factor** $b^2/4m^2$ $[1/\mathrm{s}^2]$. Here, b $[\mathrm{kg/s}]$ is the **damping constant** and is a measure of how fast the oscillations decay over time.

2.4 Forced damped oscillation

$$y(t) = A_0 \sin(\omega t + \varphi_0)$$



Position $y(t)$ $[\mathrm{m}]$ at **time** t $[\mathrm{s}]$ of a **sinusoidally forced, linearly damped** harmonic oscillator which has been oscillating for a while. Here A_0 $[\mathrm{m}]$ is the **amplitude** (maximum deflection), ω $[\mathrm{rad/s}]$ is the **excitation angular frequency** at which the oscillator oscillates, and φ_0 $[\mathrm{rad}]$ is a **phase constant** that defines the deflection $y(0)$ at time $t = 0$.

$$A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0{}^2)^2 + \frac{b^2\omega^2}{m^2}}}$$

Amplitude A_0 $[\mathrm{m}]$ of a sinusoidal forced harmonic oscillation as a function of the **excitation frequency** ω $[\mathrm{rad/s}]$. The amplitude decreases with the **damping constant** b $[\mathrm{kg/s}]$. Here ω_0 $[\mathrm{rad/s}]$ is the **natural frequency** of the oscillator, m $[\mathrm{kg}]$ the **mass** of the oscillator and F_0 $[\mathrm{N}]$ the **maximum excitation force**: $F = F_0 \cos(\omega t)$.

$$\varphi_0 = \arctan\left(\frac{\omega_0{}^2 - \omega^2}{\omega b} m\right)$$

Phase angle φ_0 $[\mathrm{rad}]$ of a sinusoidal forced damped oscillation indicates the deflection $y(t)$ at time $t = 0$. Here ω $[\mathrm{rad/s}]$ is the **excitation frequency**, which is in the sinusoidal excitation force: $F = F_0 \cos(\omega t)$.

2.5 Standing waves and reflection of waves at fixed and loose ends

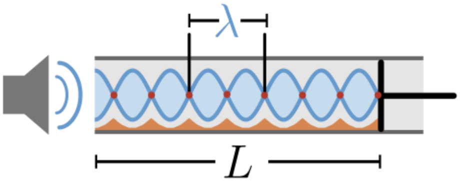
$$x_\mathrm{b} = \frac{\lambda}{2}\left(n + \frac{1}{2}\right) \qquad x_\mathrm{k} = \frac{n\lambda}{2}$$

If a wave is reflected at a *fixed* end (wall), the reflected wave undergoes a *phase jump* of 180 degrees. The incoming wave and the reflected wave overlap and a *standing wave* with non-oscillating nodes and oscillating antinodes is created. The **distance** x_{b} [m] of an oscillation antinode to the wall and the **distance** x_{k} [m] of a node to the wall depend on the **wavelength** λ [m] or on the **frequency** f [Hz]. Here $n = 0, 1, 2, \ldots$ is an **integer** numbering the nodes and and c [m/s] is the **speed of light**.

If, on the other hand, the wave is reflected at a *loose* end, then the above formulas for nodes and antinodes apply the other way round and there is no phase jump of the reflected wave.

2.6 Standing waves in a tube open on one side (Kundt's tube)

$$L = \frac{\lambda}{4}(2n - 1) \qquad L = \frac{c}{4f}(2n - 1)$$



An acoustic signal (e.g. from a loudspeaker) is passed through one end of an air-filled tube. The other end of the tube is closed with a (fixed or movable) piston. For a standing acoustic wave to form in the tube, the tube must have only a certain **length** L [m], which depends on the **wavelength** λ [m] or the **frequency** f [Hz] of the acoustic signal. Here, $n = 0, 1, 2, \ldots$ is a **natural number** indicating all possible pipe lengths and c [m/s] is the **speed of light**.

2.7 (Acoustic) beat

$$y_{\text{r}}(t) = y_0 \cos\left(2\pi \frac{f_1 - f_2}{2} t\right) \sin\left(2\pi \frac{f_1 + f_2}{2} t\right)$$

$$f_{\text{r}} = \frac{f_1 + f_2}{2} \qquad f_{\text{s}} = \frac{f_1 - f_2}{2}$$

If two sinusoidal signals with the same **amplitude** y_0 [−] and slightly different **frequencies** f_1 [Hz] and f_2 [Hz] are overlapped, the resulting signal has the **amplitude** $y_{\text{r}}(t)$ [−] and the **resulting frequency** f_{r} [Hz] as well as a **beat frequency** f_{s} [Hz].

2.8 Sound

$$c_{\text{s}} \approx \left(331.6 + 0.6 \frac{T}{°\text{C}}\right) \frac{\text{m}}{\text{s}}$$

The **speed of sound** c_s $[\mathrm{m/s}]$ in air depends on the **air temperature** T $[°\mathrm{C}]$. The formula is considered a good approximation for temperatures between -20 °C and 40 °C. The speed of sound also depends on the propagation medium. See the following table:

Medium at 20 °C		Speed of sound c_s in $\mathrm{m/s}$
Air	Gas	344
Helium	Gas	981
Benzene	Liquid	1326
Water	Liquid	1484
Plexiglass	Solid	2670
Iron	Solid	5170

$$v = \frac{\Pi_\mathrm{rms}}{\rho c_\mathrm{s}} \qquad v = \frac{I}{\Pi_\mathrm{rms}} \qquad v = \sqrt{\frac{I}{\rho c_\mathrm{s}}} \qquad v = \sqrt{\frac{P}{\rho c_\mathrm{s} A}} \qquad v = \sqrt{\frac{w}{\rho}}$$

Particle velocity v $[\mathrm{m/s}]$ is the instantaneous velocity of a vibrating air particle (or other gas) during sound propagation. Here $Z = \rho c_\mathrm{s}$ $[\mathrm{Ns/m^3}]$ is called the **acoustic impedance**.

- Π_rms $[\mathrm{Pa}]$ is the **effective sound pressure** and describes a variation of the static pressure (for example air pressure).
- I $[\mathrm{W/m^2}]$ is the **sound intensity** and describes the **sound power** P $[\mathrm{W}]$ per **transmitted area** A $[\mathrm{m^2}]$.
- w $[\mathrm{J/m^3}]$ is the **sound energy density** and describes the **sound energy** W $[\mathrm{J}]$ per volume at a point in space.
- ρ $[\mathrm{kg/m^3}]$ is the **mass density** (for example air density) and describes number of gas particles per volume.

$$L_\mathrm{p} = 20 \cdot \log_{10}\left(\frac{\Pi_\mathrm{rms}}{\Pi_0}\right) \qquad \frac{\Pi_\mathrm{rms}}{\Pi_0} = 10^{\frac{L_\mathrm{p}}{20}}$$

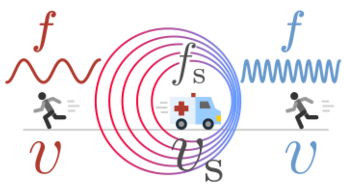
$$L_\mathrm{p} = 10 \cdot \log_{10}\left(\frac{I_1}{I_0}\right) \qquad \frac{I_1}{I_0} = 10^{\frac{L_\mathrm{p}}{10}}$$

The **sound pressure level** (SPL) L_p $[\mathrm{dB}]$ in decibels is a logarithmic measure for the sound intensity I. Here Π_rms $[\mathrm{Pa}]$ is the **effective sound pressure** and $\Pi_0 = 2 \cdot 10^{-5}\ \mathrm{Pa}$ is the **sound pressure reference value** for the sound level and indicates the hearing threshold of the human ear at a sound frequency of $1\mathrm{kHz}$. $I_0 = 10^{-12}\ \mathrm{W/m^2}$ is the **intensity reference value** for the sound level.

Sound source	Distance to sound source	Π_{rms}	L_{p}
Jet aircraft	30 m	630 Pa	150 dB
Pain threshold of the ear	Directly at the ear	100 Pa	134 dB
Limit for hearing damage (short-term exposure)	Directly at the ear	20 Pa	120 dB
Pneumatic hammer	1 m	2 Pa	100 dB
Limit for hearing damage (long-term exposure)	Directly at the ear	0.36 Pa	85 dB
Ordinary conversation	1 m	0.01 Pa	54 dB

2.9 Acoustic Doppler Effect

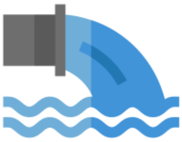
$$f = f_{\mathrm{s}}\left(\frac{c + v}{c - v_{\mathrm{s}}}\right) \quad f' = f_{\mathrm{s}}\left(\frac{c - v}{c + v_{\mathrm{s}}}\right)$$



Frequency f [Hz] of a signal (e.g. from the siren of an ambulance) perceived by an observer moving at **speed** v [m/s] when the transmitter (ambulance) moves *towards* the observer at **speed** v_{s} [m/s]. The observer perceives the **frequency** f' [Hz] when the transmitter moves *away* from the observer moving at **speed** v [m/s]. Here, c [m/s] is the **speed of sound** and f_{s} [Hz] is the **transmitter frequency** perceived by the transmitter itself.

3. FLUID DYNAMICS

Physics of liquids and gases.



3.1 Air resistance (drag)

$$F = \frac{1}{2} A c_{\mathrm{w}} \rho v^2$$

If a body of **cross-sectional area** A $[\mathrm{m}^2]$ moves with **velocity** v $[\mathrm{m/s}]$ in air of **density** ρ $[\mathrm{kg/m}^3]$, the air **resistance force** F $[\mathrm{N}]$ acts in the opposite direction to the motion on the body. Here, the constant c_{w} $[-]$ is the **drag coefficient**, which depends on the shape of the body.

Body shape	Drag coefficient c_{w}
Long rectangle plate	2
Long cylinder	1.2
Disc	1.12
Sphere	0.45
Human	≈ 0.8
Car	0.15 - 0.7
Airplane	≈ 0.1

3.2 Compressibility and bulk modulus

$$k = -\frac{1}{V}\frac{\Delta V}{\Delta \Pi} \qquad Q = \frac{1}{k}$$

Compressibility k $[1/\mathrm{Pa}]$ of a fluid whose **volume** V $[\mathrm{m}^3]$ decreases by the value $\Delta V = V_1 - V$ $[\mathrm{m}^3]$ when the fluid undergoes a small **pressure change** $\Delta \Pi = \Pi_1 - \Pi$ $[\mathrm{Pa}]$. Compressibility indicates how easily a material can be compressed. The **bulk modulus** Q $[\mathrm{Pa}]$ is the reciprocal of the compressibility.

Material		Compressibility k in $1/\mathrm{Pa}$	Bulk modulus Q in Pa
Air	Gas	10^{-5}	10^{5}
Ethanol	Liquid	$1.12 \cdot 10^{-9}$	$0.896 \cdot 10^{9}$
Acetone	Liquid	$1.09 \cdot 10^{-9}$	$0.92 \cdot 10^{9}$
Sodium (Na)	Solid	$0.16 \cdot 10^{9}$	$6.3 \cdot 10^{9}$
Aluminum (Al)	Solid	$0.013 \cdot 10^{9}$	$76 \cdot 10^{9}$
Gold (Au)	Solid	$0.006 \cdot 10^{9}$	$180 \cdot 10^{9}$
Diamond	Solid	$0.002 \cdot 10^{9}$	$442 \cdot 10^{9}$

3.3 Surface tension

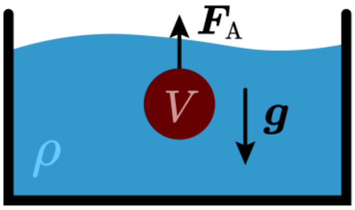
$$\sigma = \frac{W_{\mathrm{s}}}{A}$$

Surface tension σ $[\mathrm{J/m^2} = \mathrm{N/m}]$ of a liquid is defined as **surface energy** W_S [J] per **surface area** A $[\mathrm{m^2}]$ of the liquid.

Liquid	Surface tension σ at 20 °C in $\mathrm{J/m^2}$
Ethanol	$22.6 \cdot 10^{-3}$
Aceton	$23.3 \cdot 10^{-3}$
Benzol	$28.9 \cdot 10^{-3}$
Glycerin	$63.4 \cdot 10^{-3}$
Wasser	$72.8 \cdot 10^{-3}$
Quecksilber (Hg)	$476.0 \cdot 10^{-3}$

3.4 Buoyancy force (Archimedes' principle)

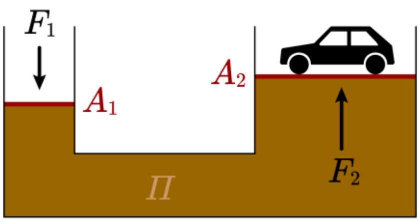
$$F_\mathrm{A} = \rho V g$$



Buoyant force F_A [N] experienced by a body of **volume** V $[\mathrm{m^3}]$ in the opposite direction to the gravitational force when this body is immersed in a liquid (for example water) of **density** ρ $[\mathrm{kg/m^3}]$. It is said: the body experiences buoyancy. Here, g $[\mathrm{m/s^2}]$ is the **gravitational acceleration**.

3.5 Hydraulic press and volume work

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



The quotient of the **force** F_1 [N] on the supporting **surface** A_1 $[\mathrm{m^2}]$ is equal to the quotient of the **force** F_2 [N] on the second supporting **surface** A_2 $[\mathrm{m^2}]$ of the **hydraulic press**.

$$W = \Pi \Delta V$$

Volume work W [J] is the energy converted when the **initial volume** V_1 $[\mathrm{m^3}]$ of a gas, in which the **pressure** Π [Pa] prevails, is changed from the value V_1 to the value V_2 $[\mathrm{m^3}]$. The volume changes by the value $\Delta V = V_2 - V_1$.

- If ΔV is **negative**, then the fluid is doing the work **on the piston**.
- If ΔV is **positive**, then the piston is doing the work **on the fluid**.

3.6 Flow (viscosity)

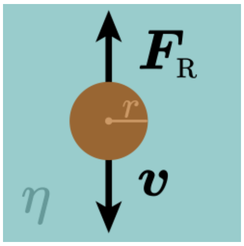
$$\eta = \frac{Fd}{v_0 A}$$

Viscosity η $[\mathrm{Pa \cdot s} = \mathrm{kg/(m \cdot s)}]$ of a liquid between two parallel plates with **inner surface** A $[\mathrm{m^2}]$ and **distance** d $[\mathrm{m}]$ from each other. A **force** F $[\mathrm{N}]$ is applied parallel to one plate, resulting in a **shear stress** F/A $[\mathrm{N/m^2}]$ on the fluid. The fluid next to the moving plate has a **velocity** v_0 $[\mathrm{m/s}]$.

Liquid	Viscosity η at 20 °C in $\mathrm{Pa \cdot s}$
Water	$1.008 \cdot 10^{-3}$
Olive oil	$108 \cdot 10^{-3}$
Glycerin	$1500 \cdot 10^{-3}$
Honey	$10\,000 \cdot 10^{-3}$
Tar	$100\,000 \cdot 10^{-3}$

3.7 Viscous friction (Stokes' law)

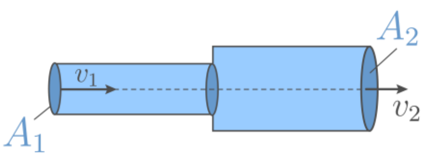
$$F_{\mathrm{R}} = 6\pi\eta r v \qquad \mu = 6\pi\eta r$$



If a **spherical particle** with **radius** r $[\mathrm{m}]$ moves in a fluid (for example water or air) of **viscosity** η $[\mathrm{Pa \cdot s} = \mathrm{kg/(m \cdot s)}]$ with **velocity** v $[\mathrm{m/s}]$ downwards, then the particle experiences a Stokes **friction force** F_{R} $[\mathrm{N}]$ in the opposite direction of motion. Here μ $[\mathrm{kg/s}]$ is called the **friction coefficient**.

3.8 Continuity equation

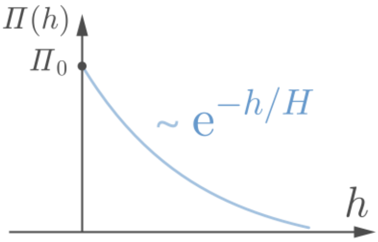
$$A_1 v_1 = A_2 v_2$$



A fluid flows through a pipe with **cross-sectional area** A_1 $[\mathrm{m^2}]$ at a certain point and with **flow velocity** v_1 $[\mathrm{m/s}]$. At another point in the pipe, the **cross-sectional area** is A_2 $[\mathrm{m^2}]$. The flow velocity there has changed to v_2 $[\mathrm{m/s}]$.

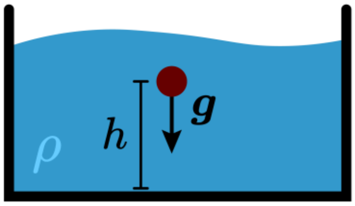
3.9 Air pressure and hydrostatic gravity pressure of a fluid at rest

$$\Pi(h) = \Pi_0 e^{-h/H}$$



Static pressure Π $[\mathrm{Pa}]$ (for example, air pressure) of a gas at **height** h $[\mathrm{m}]$ above the sea level. The (mean) **air pressure at sea level** at $h = 0$ is approximately $\Pi_0 = 101\mathrm{kPa} = 1\mathrm{bar}$. Here H $[1/\mathrm{m}]$ is called **scale height** and depends on *temperature* and *gravitational acceleration*. This formula is called *barometric formula*.

$$\Pi_{\mathrm{hyd}} = \rho g h + \Pi_0$$

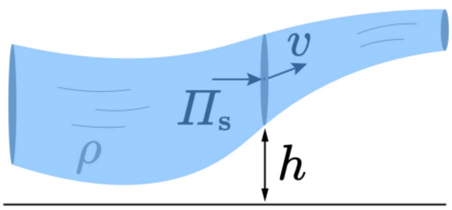


Hydrostatic pressure Π_{hyd} $[\mathrm{Pa}]$ is the pressure of a fluid with **density** ρ $[\mathrm{kg/m^3}]$ at **depth** h $[\mathrm{m}]$ or of the gas at **height** h. In the case of fluids the density ρ is constant, in the case of a gas ρ is not constant. Here Π_0 is the **pressure at** $h = 0$ (for example at sea level) and g $[\mathrm{m/s^2}]$ is the **gravitational acceleration**.

3.10 Bernoulli equation and the dynamic fluid

$$\Pi = \Pi_{\mathrm{s}} + \Pi_{\mathrm{dyn}} + \Pi_{\mathrm{hyd}}$$

$$\Pi = \Pi_{\mathrm{s}} + \frac{1}{2}\rho v^2 + \rho g h$$



Total pressure Π $[\mathrm{Pa} = \mathrm{N/m}]$ of a stationary, non-viscous, incompressible fluid along a streamline. The total pressure here is the sum of the **dynamic pressure** $\Pi_{\mathrm{dyn}} = \frac{1}{2}\rho v^2$ $[\mathrm{Pa}]$, **hydrostatic pressure** $\Pi_{\mathrm{hyd}} = \rho g h$ $[\mathrm{Pa}]$ and **static pressure** Π_{s} $[\mathrm{Pa}]$. Where ρ $[\mathrm{kg/m^3}]$ is the **density** of the fluid, h $[\mathrm{m}]$ is the **height** of the considered streamline of a fluid above the ground (or other specified zero point), and v $[\mathrm{m/s}]$ is the **flow velocity** of the fluid.

3.11 Volumetric flow rate and flow resistance

$$Q = \frac{\Delta V}{\Delta t} \qquad R = \frac{\Delta \Pi}{Q}$$

A **volumetric flow rate** Q $[\mathrm{m^3/s}]$ indicates the **volume** ΔV $[\mathrm{m^3}]$ of liquid which has passed through a certain cross-sectional area (e.g. of a pipe) within the **time** Δt [s].

Flow resistance R $[\mathrm{Ns/m^5}]$ (for example *vascular resistance*) is the **pressure difference** $\Delta\Pi = \Pi_1 - \Pi_2$ $[\mathrm{Pa}]$ between two ends of a pipe per volume flow Q.

3.12 Reynolds number

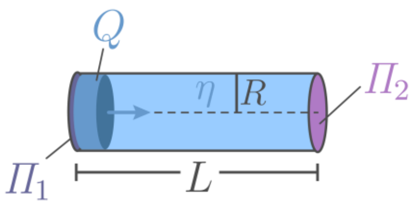
$$R_e = \frac{\rho v L}{\eta}$$

The **Reynolds number** R_e $[-]$ is a measure of how *turbulent* a flow is. The Reynolds number depends on the **density** ρ $[\mathrm{kg/m^3}]$ of the **flow velocity** v $[\mathrm{m/s}]$, on the **dynamic viscosity** η $[\mathrm{Pa \cdot s = kg/(m \cdot s)}]$ and on the **characteristic length** L $[\mathrm{m}]$ of the body (for example radius of a pipe) in which the fluid moves. Above a critical value of about $R_e = 1100$, the flow usually becomes turbulent.

3.13 Hagen–Poiseuille equation

$$Q = \frac{\pi R^4 (\Pi_2 - \Pi_1)}{8 \eta L}$$

$$Q = \frac{A^2 (\Pi_2 - \Pi_1)}{8 \pi \eta L}$$



A **volumetric flow rate** Q $[\mathrm{m^3/s}]$ of a fluid of **viscosity** η $[\mathrm{Pa \cdot s = Ns/m}]$ through a pipe of **radius** R $[\mathrm{m}]$, **length** L $[\mathrm{m}]$ and **cross-sectional area** A $[\mathrm{m^2}]$ occurs due to a **pressure difference** $\Delta\Pi = \Pi_2 - \Pi_1$ $[\mathrm{Pa}]$ between the beginning and end of the pipe.

4. THERMODYNAMICS

Physics of heat transfer and energy conversion.

4.1 Relationship between Kelvin and Celsius temperature

$$T = 273.15\mathrm{K} + 1\frac{\mathrm{K}}{°\mathrm{C}} \cdot \vartheta \qquad \Delta T = T_2 - T_1 = \Delta\vartheta$$

Absolute temperature T [K] is the **Celsius temperature** ϑ [°C] shifted by 273.15 Kelvin. The *differences* ΔT [K] and $\Delta\vartheta$ [°C] of two Kelvin ($_1$ and T_2) or Celsius (ϑ_1 and ϑ_2) temperatures are equal.

4.2 Efficiency

$$\eta = \frac{P_+}{P_-} \qquad \eta = \frac{W_+}{W_-}$$

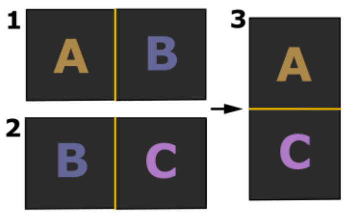
Efficiency η [−] tells how effectively a machine can convert one form of energy (for example *mechanical* energy) into another form of energy (for example *electrical* energy). The efficiency is the ratio of the **power** P_- [W] or **energy** W_- [J] **supplied** to the system and the **usable power** P_+ [W] or **usable energy** W_+ [J] obtained from the system.

$$\eta_\mathrm{c} = 1 - \frac{T_-}{T_+} \qquad \eta_\mathrm{c} = \frac{Q_- + Q_+}{Q_-} \qquad \eta_\mathrm{c} = \frac{|W|}{Q_-}$$

Carnot efficiency η_c [−] (**thermodynamic efficiency**) is the theoretically *maximum* achievable efficiency when converting the **heat energy** Q_- [J] into (**mechanical**) **energy** $|W|$. Here, T_- [K] is the **lowest temperature** and T_+ [K] is the **highest temperature** occurring in the Carnot process. In other words, Q_+ [J] is the **dissipated heat energy** gained from the Carnot process at the absolute temperature T_+ and Q_- [J] is the **supplied heat energy** at the absolute temperature T_-. The (**realizable**) **efficiency** η [−] is always smaller than the Carnot efficiency: $\eta < \eta_\mathrm{c}$.

4.3 Laws of thermodynamics

Zeroth law of thermodynamics



If two **systems**, for example **A** and **B**, are **in thermal equilibrium**, then they have the equal temperatures: $T_\mathrm{A} = T_\mathrm{B}$. If the temperature equality characterizes a thermal equilibrium, then we can conclude from the zeroth law:

$$\text{If } T_\mathrm{A} = T_\mathrm{B} \text{ and } T_\mathrm{B} = T_\mathrm{C} \text{ then } T_\mathrm{A} = T_\mathrm{C}$$

First law of thermodynamics (energy conservation)

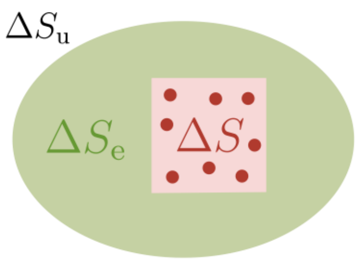
$$\Delta U = \Delta Q + \Delta W$$

Change in internal energy $\Delta U = U_2 - U_1$ [J] of a system (for example, a gas) is the sum of the **thermal energy** (**heat**) ΔQ [J] **supplied to** the system ($\Delta Q > 0$) or **released from** the system ($\Delta Q < 0$) and the **work** ΔW [J] done **on** the system ($\Delta W > 0$) or **by** the system ($\Delta W < 0$).

$$\Delta W = -\Pi \Delta V$$

Volume work ΔW [J] done on or by the system when the **volume** has changed by ΔV [$\mathrm{m^3}$] of the system due to the **external pressure** Π [Pa] acting on the system.

Second law of thermodynamics



$$\Delta S_\mathrm{u} = \Delta S + \Delta S_\mathrm{e} > 0$$

The **entropy change** ΔS_u [J/K] **of the universe** is composed of the **entropy change of a system** ΔS [J/K] and the **entropy change** ΔS_e [J/K] **of its surroundings**. The entropy change ΔS_u of the universe is always positive during spontaneous processes.

Third law of thermodynamics (Nernst theorem)

Entropy change ΔS of a closed system at $T = 0\mathrm{K}$ **is zero**: $\Delta S(T = 0\mathrm{K}) = 0$.
Or equivalently: The **entropy** S of a system at $T = 0\mathrm{K}$ **is constant**: $S = \mathrm{const.}$
Or: The **absolute zero** $T = 0\mathrm{K}$ is **not reachable**.

4.4 Specific and molar gas constant

$$R_\mathrm{s} = \frac{R}{M_\mathrm{n}} \qquad R_\mathrm{s} = \frac{k_\mathrm{B}}{m} \qquad R_\mathrm{s} = c_\Pi - c_\mathrm{V}$$

$$R = N_\mathrm{A} k_\mathrm{B} = 8.314\,462\,618\,153\,24\ \frac{\mathrm{J}}{\mathrm{mol \cdot K}}$$

The **specific gas constant** R_s[J/kg · K] refers to the mass of the gas and indicates the ratio of the **molar gas constant** R [J/mol · K] to the **molar mass** $M_\mathrm{n} = M/n$ [kg/mol] of the gas.

Alternatively, it can be calculated using the **mass m** [kg] **of a gas particle** and **Boltzmann constant k_B** or experimentally from the difference of **specific heat capacity c_Π** [J/kg · K] (constant *pressure*) and **specific heat capacity c_V** [J/kg · K] (constant *volume*).

Gas	Specific gas constant R	Molar mass M
Helium (He)	2077.1 J/kg · K	$4.003 \cdot 10^{-3}$ kg/mol
Methane (CH_4)	518.4 J/kg · K	$16.04 \cdot 10^{-3}$ kg/mol
Nitrogen (N_2)	296.8 J/kg · K	$28.01 \cdot 10^{-3}$ kg/mol
Oxygen (O_2)	259.8 J/kg · K	$32.00 \cdot 10^{-3}$ kg/mol
Carbon dioxide (CO_2)	188.9 J/kg · K	$44.01 \cdot 10^{-3}$ kg/mol

4.5 Mass concentration, specific/molar volume and molality

$$\rho_n = \frac{n}{V} \qquad V_s = \frac{V}{m} = \frac{1}{\rho} \qquad V_n = \frac{V}{n} \qquad c_n = \frac{n}{m}$$

- **Molarity** (amount of substance concentration) ρ_n [mol/m³] is **amount of substance n** [mol] per **volume** V [m³].
- **Specific volume** V_s [m³/kg] is **volume** V per **mass** m [kg].
- **Molar volume** V_n [m³/mol] is **volume** V per **amount of substance n**.
- **Molality** c_n [mol/kg] is **amount of substance n** per **mass** m.

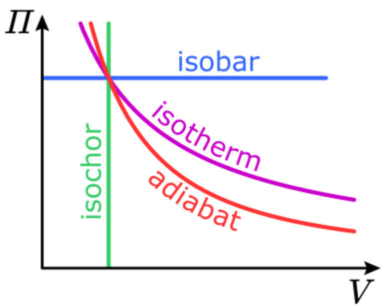
4.6 Ideal gas

$$\Pi V = nRT \qquad \Pi V = N k_B T$$

$$\Pi V = \frac{1}{3} N m \overline{v^2} \qquad \Pi V = M R_s T$$

$$\Pi = \frac{1}{3} \rho \overline{v^2} \qquad \Pi = \rho R_s T$$

$$\Pi = \rho_N k_B T$$



The **ideal gas** is in a closed system with **volume** V [m³], **pressure** Π [Pa = kg/m · s²] and **temperature** T [K]. Here, R [J/mol · K] is the **molar gas constant**, R_s [J/kg · K] is the **specific gas constant**, and $n = N/N_A$ [mol] is the **amount of substance**, which is the ratio of the **number N** [−] **of gas particles** to the Avogardo constant. Here, m is the **mass of a gas particle**, and $\overline{v^2}$ [m²/s²] is the **mean square of the velocity** of the gas particles.

The gas pressure Π can also be calculated using the **mass density** $\rho = M/V$ $[\mathrm{kg/m^3}]$ (**total mass** $M = Nm$ of the gas per gas **volume** V) or using the **particle density** $\rho_N = N/V$ $[1/\mathrm{m^3}]$.

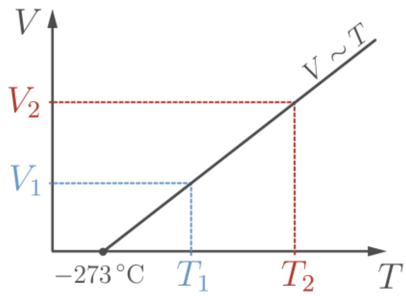
$$N = nN_\mathrm{A}$$

Number of gas particles N $[-]$ can be calculated from the **amount of substance** n [mol], if the amount of substance is multiplied by the **Avogadro constant** N_A [1/mol].

$$W_\mathrm{kin} = \frac{3}{2}k_\mathrm{B}T$$

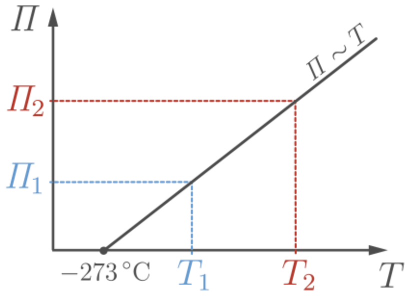
Kinetic energy W_kin [J] of a gas particle depends on the **temperature** T [K] of the ideal gas.

$$\frac{T_1}{V_1} = \frac{T_2}{V_2}$$



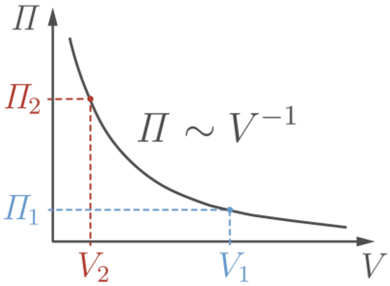
Gay-Lussac law describes the linear relationship between the **volume** V $[\mathrm{m^3}]$ and the **temperature** T [K, °C] of an ideal gas at **constant pressure.** Here, V_1 is the **volume** at **temperature** T_1 and the **volume** V_2 at **temperature** T_2.

$$\frac{\Pi_1}{T_1} = \frac{\Pi_2}{T_2}$$



Amonton's law describes the linear relationship between the **pressure** Π [Pa] and the **temperature** T [K, °C] of an ideal gas at **constant volume**. Here Π_1 is the **pressure** at **temperature** T_1 and the **pressure** Π_2 at **temperature** T_2.

$$\frac{\Pi_1}{\Pi_2} = \frac{V_2}{V_1}$$



Boyle-Mariotte law describes the relationship between the **pressure** Π [Pa] and **volume** V [K, °C] of an ideal gas at a **constant temperature.**

4.7 Real gas (Van der Waals equation)

$$\Pi = \frac{nRT}{V - nV_n} \frac{n^2 a}{V^2}$$

The **Van der Waals equation** describes the **pressure** Π $[\mathrm{Pa} = \mathrm{kg/m \cdot s^2}]$ of a *real* gas as a function of the **temperature** T $[\mathrm{K}]$ of the gas and of its **volume** V $[\mathrm{m^3}]$. For a real gas, the pressure may be very high and the temperature very small. Here $n = \frac{N}{N_A}$ $[\mathrm{mol}]$ is the **amount of substance** and R is the **gas constant**. V_n $[\mathrm{m^3/mol}]$ is the **covolume** of the gas, the volume available for motion reduced by the value nV_n. For the ideal gas, $V_n = 0$. And a $\left[\mathrm{Pa \cdot m^6/mol^2}\right]$ is the **cohesion parameter** - a material-dependent constant that takes into account the force effect between the gas particles. For the ideal gas, $a = 0$.

4.8 Entropy / enthalpy

$$\Delta S = C \ln\left(\frac{T_2}{T_1}\right) \qquad \Delta S = nR \ln\left(\frac{\Pi_2}{\Pi_1}\right)$$

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right) \qquad \Delta S = mR_s \ln\left(\frac{\Pi_2}{\Pi_1}\right)$$

$$\Delta S = mR_s \ln\left(\frac{V_2}{V_1}\right)$$

The **entropy change** $\Delta S = S_2 - S_1$ $[\mathrm{J/K}]$ of an ideal gas from the **initial value** S_1 to the **final value** S_2:

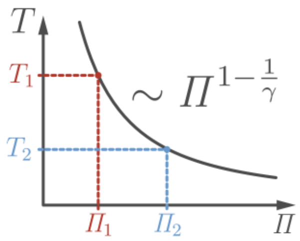
- Due to an *isobaric/isochoric* change of **temperature** T_1 $[\mathrm{K}]$ to T_2.
- Due to an *isothermal* change of the pressure Π_1 $[\mathrm{Pa}]$ to Π_2.
- Due to an *isothermal* change of the **volume** V_1 $[\mathrm{m^3}]$ to V_2.

Here it is assumed that the **heat capacity** C $[\mathrm{J/K}]$ (C_Π or C_V) is independent of temperature. $R = N_A k_B$ $[\mathrm{J/mol \cdot K}]$ is the **gas constant** and $n = N/N_A$ $[\mathrm{mol}]$ is the **amount of substance**, which can be calculated as the ratio of the **number N of gas particles** and the **Avogadro constant** N_A.

4.9 Adiabatic (without heat exchange) process

$$T_1 \Pi_1^{\frac{1}{\gamma}-1} = T_2 \Pi_2^{\frac{1}{\gamma}-1} \quad \Pi_1 V_1^{\gamma} = \Pi_2 V_2^{\gamma}$$

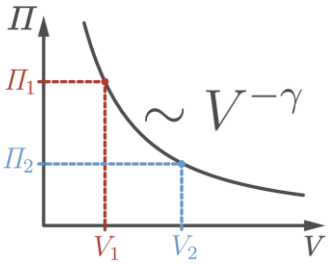
$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

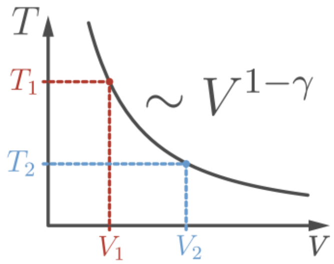


The product of **temperature** T_1 [K] and **pressure** Π_1 [Pa] with **adiabatic exponent** γ [−] *before* adiabatic process is equal to **pressure** Π_2 and **temperature** T_2 *after* adiabatic process. The above adiabatic equations can be rearranged with respect to adiabatic exponent:

$$\gamma = \left(\frac{\ln(T_1) - \ln(T_2)}{\ln(\Pi_2) - \ln(\Pi_1)} + 1\right)^{-1} \quad \gamma = \frac{\ln(\Pi_1) - \ln(\Pi_2)}{\ln(V_2) - \ln(V_1)}$$

$$\gamma = \frac{\ln(T_1) - \ln(T_2)}{\ln(V_2) - \ln(V_1)} + 1$$

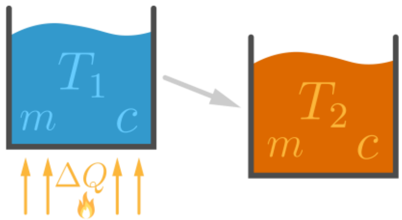




4.10 Heat energy and heat capacity

$$\Delta Q = cm\Delta T \quad C = cm$$

$$C_\mathrm{n} = \frac{C}{n}$$



Thermal energy (**heat energy**) ΔQ [J] is the energy *supplied* to or *released* from a substance of **mass** m [kg] when it is changed from the **initial temperature** T_1 [K] to the **final temperature** T_2 [K]: $\Delta T = T_2 - T_1$ [K]. This thermal energy also depends on the **specific heat capacity** c [J/kg K] of the substance. Here, C [J/K] is the **heat capacity** and C_n [J/mol · K] is the **molar heat capacity**.

Material		Specific heat capacity c in $\mathrm{J/kg \cdot K}$
Helium (He)	Gas	5190
Water	Liquid	4180
Ice (0 °C)	Solid	2060
Air	Gas	1010
Aluminium (Al)	Solid	900
Lead (Pb)	Solid	129

$$W_\mathrm{s} = c_\mathrm{s} m \quad W_\mathrm{v} = c_\mathrm{v} m$$

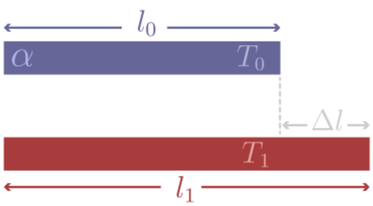
For a phase transition (for example from solid to liquid) an additional energy is required:

- If a substance of **mass** m [kg] is *melted*, then the **melting energy** (**heat of fusion**) W_s [J] is required for the phase transition (solid → liquid).
- If a substance of **mass** m [kg] is *vaporized*, then the **evaporation energy** (**heat of vaporization**) W_v [J] is necessary for the phase transition (liquid → gaseous or solid → gaseous).

Material		Spec. melting energy c_s in $\mathrm{J/kg}$	Spec. evaporation energy c_v in $\mathrm{J/kg}$	Boiling temperature
Aluminium (Al)	Solid	$398 \cdot 10^3$	$10500 \cdot 10^3$	2743 K / 2470°C
Lead (Pb)	Solid	$25 \cdot 10^3$	$871 \cdot 10^3$	2022 K / 1749°C
Ice	Solid	$334 \cdot 10^3$	-	-
Gold (Au)	Solid	$63 \cdot 10^3$	$1578 \cdot 10^3$	2973 K / 2700°C
Water	Liquid	-	$2256 \cdot 10^3$	373 K / 100°C
Iron (Fe)	Solid	$268 \cdot 10^3$	$6322 \cdot 10^3$	3135 K / 2862°C

4.11 Thermal expansion in length and volume

$$\Delta l = l_0 \alpha (T_1 - T_0)$$



Length Δl [m] by which a metallic body (for example a metal rod) has changed after heating/cooling. Here l_0 [m] is the **initial length** before the temperature change and α [1/K] is the **coefficient of thermal expansion**, which depends on the material from which the body is made. The body is brought from the **initial temperature** T_0 [K] to the **final temperature** T_1 [K].

$$\Delta V = V_0 \gamma (T_1 - T_0)$$

Here ΔV $[\mathrm{m^3}]$ is the volume change, V_0 $[\mathrm{m^3}]$ is the **initial volume** before the temperature change. The body was brought from the **initial temperature** T_0 $[\mathrm{K}]$ to the **final temperature** T_1 $[\mathrm{K}]$. And γ $[1/\mathrm{K}]$ is the **coefficient of volume expansion**. For *isotropic* bodies, $\gamma = 3\alpha$.

Material at 20°C		Coefficient of thermal expansion α in $1/\mathrm{K}$	Coefficient of volume expansion γ in $1/\mathrm{K}$
Aluminium (Al)	Solid	$23.1 \cdot 10^{-6}$	$69.3 \cdot 10^{-6}$
Lead (Pb)	Solid	$28.9 \cdot 10^{-6}$	$86.7 \cdot 10^{-6}$
Iron (Fe)	Solid	$11.8 \cdot 10^{-6}$	$35.4 \cdot 10^{-6}$
Gold (Au)	Solid	$14.2 \cdot 10^{-6}$	$42.6 \cdot 10^{-6}$
Wood (oak)	Solid	$8 \cdot 10^{-6}$	-
Concrete	Solid	$12 \cdot 10^{-6}$	$36 \cdot 10^{-6}$
Water (H2O)	Liquid	-	$0.21 \cdot 10^{-3}$
Mercury (Hg)	Liquid	-	$0.18 \cdot 10^{-3}$

4.12 Chemical reactions

$$k(T) = A(T)\mathrm{e}^{-\frac{W_\mathrm{A}}{RT}} \qquad A(T) = \sigma\sqrt{\frac{9k_\mathrm{B}T}{\pi m^*}}N_\mathrm{A}$$

The **Arrhenius equation** states that the **reaction rate constant** k $[\mathrm{m^3/mol \cdot s}]$ (a measure of the rate of a chemical reaction) depends approximately *exponentially* on the **temperature** T $[\mathrm{K}]$. The **frequency factor** $A(T)$ $[\mathrm{m^3/mol \cdot s}]$ is also slightly temperature dependent and can be calculated according to collision theory using the **collisional cross section** σ $[\mathrm{m^2}]$ and **reduced mass** m^* $[\mathrm{kg}]$. Here, W_A $[\mathrm{J/mol}]$ is the **activation energy** required for a chemical reaction. R is the **gas constant**, N_A is the **Avogadro constant**, and k_B is the **Boltzmann constant**. The exponent is called the **Arrhenius number** $\gamma = -W_\mathrm{A}/RT$ $[-]$.

$$W_\mathrm{A} = R\frac{T_1T_2}{T_2 - T_1}\ln\left(\frac{k_2}{k_1}\right)$$

The **activation energy** W_A $[\mathrm{J/mol}]$ can be determined using *two* **reaction rate constants** $k_1(T_1)$ and $k_2(T_2)$ at two different **temperatures** T_1 and T_2.

4.13 Fick's laws of diffusion

$$J \approx -D\frac{\Delta c}{\Delta x} \qquad J \approx -P\Delta c$$

The **Fick's laws of diffusion** describes the **particle current density** J $[\mathrm{mol/s \cdot m^2}]$, which arises due to a change of the **particle concentration** $\Delta c = c_2 - c_1$ $[\mathrm{mol/m^3}]$ along a short **distance** $\Delta x = x_2 - x_1$ $[\mathrm{m}]$ (for example: thickness of a membrane wall). Here, D $[\mathrm{m^2/s}]$ is the **diffusion coefficient**. The ratio $P = D/\Delta x$ $[\mathrm{m/s}]$ is called the **permeability coefficient**.

4.14 Osmotic pressure (van't Hoff equation)

$$\Pi_{\mathrm{osm}} \approx \frac{n}{V} RT$$

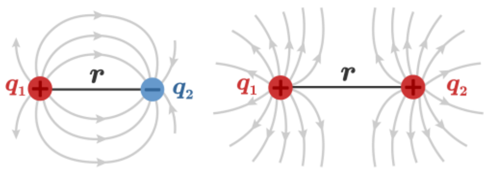
The **osmotic pressure** Π_{osm} $[\mathrm{Pa}]$ that can occur during osmosis (if the membrane does not burst before), which depends on the **temperature** T $[\mathrm{K}]$ and on the **molar concentration** $c_{\mathrm{osm}} = n/V$ $[\mathrm{mol/m^3}]$ of a system (for example a biological cell). Here, n $[\mathrm{mol}]$ is the **amount of substance** and V $[\mathrm{m^3}]$ is the **volume** of the system.

5. ELECTRODYNAMICS

Electromagnetic fields and charged particles.

5.1 Electric force between two charges (Coulomb's law) and potential

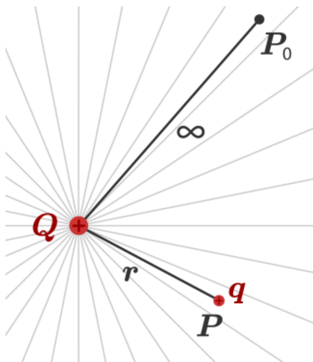
$$F_e = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$$



Electrostatic force (**Coulomb force**) F_e [N] is the *attractive* or *repulsive* electric force between two **electric charges** q_1 [C] and q_2 [C], which are located at a **distance** r [m] from each other. Here ε_0 [As/Vm] is the **vacuum permittivity** and ε_r [−] is the material dependent **relative permittivity**. The product of the two constants is called **permittivity** $\varepsilon = \varepsilon_0\varepsilon_r$ [As/Vm].

Medium	Relative permittivity ε_r
Vacuum	1 (exact)
Air (0°C)	1.0005
Glass	5 to 10
Water (0°C)	88
Water (40°C)	73.4
Ice (-20°C)	16
Hydrogen cyanide	95
Ethanol (20°C)	25.8

$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r} \qquad V = \frac{W_{pot}}{q}$$



Electric potential V [J/C] at a **distance** r [m] from a **source charge** Q [C] that generates this potential. The potential indicates the **potential energy** W_{pot} [J] that a **test charge** q [C] has gained or released when it is moved from an infinite distance to a distance r to the source charge Q.

5.2 Electric flux density, field strength and flux

$$E = \frac{F_e}{q} \qquad D = \varepsilon_0\varepsilon_r E$$

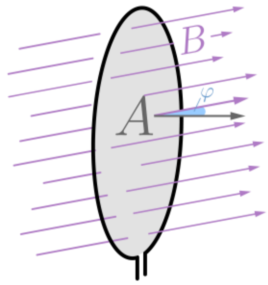
Electric field strength E [V/m] indicates the electric force that a **charge** q [C] would experience if this charge were placed in the electric field. **Electric flux density** D [C/m^2] describes the density of electric field lines per area. Here ε_r [−] is the medium-dependent **relative permittivity**.

$$\Phi_e = DA\cos(\varphi)$$

Electric flux Φ_e [As] through a **plane surface** A [m^2]. Here φ [rad] is the **angle** between the electric field lines and the *surface orthogonal vector*. If the field lines enter the surface exactly *perpendicular*, then $\varphi = \pi/2$ (90°) and the formula simplifies to: $\Phi_e = DA$.

5.3 Magnetic flux density, field strength (excitation) and flux

$$B = \mu_0 \mu_r H$$



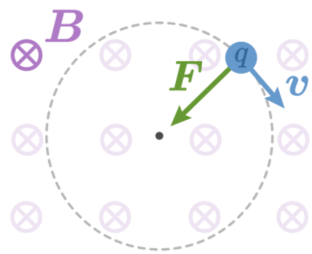
Magnetic flux density B [$T = Vs/m^2$] is in many cases *linearly* related to **magnetic field strength** H [A/m]. Here the **vacuum permeability** μ_0 [Vs/Am] and **relative permeability** μ_r [−] form the proportionality constant μ, which is called **permeability** $\mu = \mu_0 \mu_r$ [Vs/Am]. The relative permeability takes into account the medium (for example water, iron) in which the magnetic flux density is to be calculated. Attention: The formula is only valid if B and H are *parallel* to each other!

$$\Phi_m = BA\cos(\varphi)$$

Magnetic flux Φ_m [$Tm^2 = Vs$] through a **plane surface** A [m^2]. Here φ [rad] is the **angle** between the magnetic field lines and the *surface orthogonal vector*. If the field lines enter the surface *perpendicularly*, then $\varphi = \pi/2$ (90°) and the formula simplifies to: $\Phi_m = BA$.

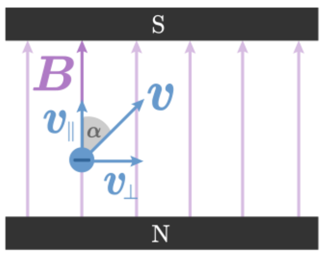
5.4 Moving charge: Lorentz force and magnetic field

$$F = qvB\sin(\alpha)$$
$$F = qvB, \quad \boldsymbol{v} \perp \boldsymbol{B}$$



An **electric charge** q [C] flying in a **magnetic field** B [T] perpendicular to its **velocity** v [m/s] experiences a **Lorentz force** F [N] (**magnetic force**). α [rad] is the **angle** between the magnetic field direction and the velocity direction.

$$r = \frac{mv}{|q|B} \qquad T = 2\pi \frac{m}{|q|B}$$



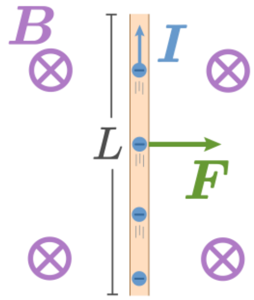
If the **charge** q with **mass** m [kg] has enough space, it will go through a circular path with **radius** r [m]. The **duration** (**period**) T [s] of a rotation depends on the **external magnetic field** B.

$$B_\mathrm{p}(r) = \frac{\mu_0 q}{4\pi} \frac{v \sin(\varphi)}{r}$$

Magnetic field $B_\mathrm{p}(r)$ [T] at **distance** r [m] from a moving particle of **charge** q [C]. This magnetic field is generated by this particle. The particle moves with **velocity** v [m/s]. Here φ [rad] is the **angle** between the velocity direction and the connecting line of length r between the particle and the location where the magnetic field is calculated.

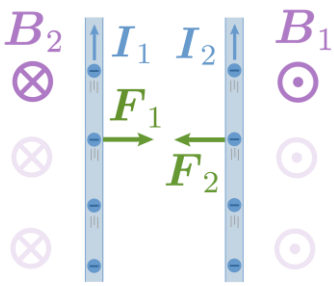
5.5 Current carrying wires: Lorentz force and magnetic field

$$F = ILB$$



A wire in which an **electric current** I [A] flows and which is located in a **magnetic field** B [T] is deflected by the **Lorentz force** F [N].

$$F = F_1 = F_2 = \frac{\mu_0 L}{2\pi} \frac{I_1 I_2}{r}$$

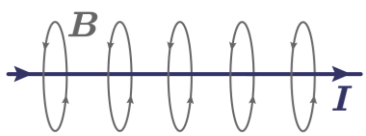


Two wires of equal **length** L [m] with **currents** I_1 [A] and I_2 [A] flowing through them are located at a **distance** r [m] from each other.

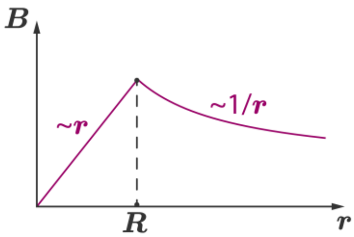
- The **current** I_1 through the *first* wire generates a **magnetic field** B_1 at the location of the *second* wire. This causes **Lorentz force** F_1 [N] to act on the *second* wire.
- The **current** I_2 through the *second* wire generates a **magnetic field** B_2 at the location of the *first* wire. This causes **Lorentz force** F_2 [N] to act on the *first* wire.

The magnitudes of the two forces F_1 and F_2 are equal. Depending on the direction of the currents, the wires *attract* or *repel* each other.

$$B_\mathrm{i}(r) = \frac{\mu_0 I}{2\pi R^2} r \qquad B_\mathrm{e}(r) = \frac{\mu_0 I}{2\pi} \frac{1}{r}$$



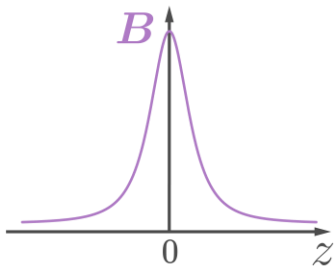
Magnetic field $B_{\text{i}}(r)$ [T] *inside* a wire at a **distance** r [m] from the longitudinal axis of the wire. The wire has the **radius** R [m] and a **current** I [A] flows through it. The magnetic field increases linearly with the distance r from the longitudinal axis of the wire.



Magnetic field $B_{\text{e}}(r)$ [T] *outside* a wire at a **distance** r [m] from the longitudinal axis of the wire. A **current** I [A] flows through the wire. The magnetic field outside decreases reciprocally with the distance r to the wire and is independent of the radius R [m] of the wire.

5.6 Magnetic field of a ring-shaped wire

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$



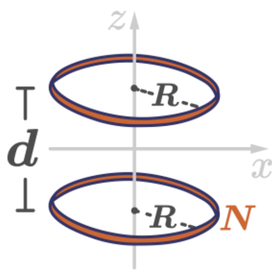
Magnetic field $B(z)$ [T] at a *perpendicular* **distance** z [m] from the center of a ring-shaped wire loop. The wire loop is perpendicular to the z-axis, where the z-axis passes through the center of the wire loop. A **current** I [A] flows through the wire loop with the **radius** R [m].

5.7 Magnetic field of a Helmholtz coil

$$B_{\downarrow\uparrow}(z) = -\frac{\mu_0 I R^2 N}{2}\left[\left(\left(z-\frac{d}{2}\right)^2 + R^2\right)^{-3/2} - \left(\left(z+\frac{d}{2}\right)^2 + R^2\right)^{-3/2}\right]$$

$$B_{\uparrow\uparrow}(z) = -\frac{\mu_0 I R^2 N}{2}\left[\left(\left(z-\frac{d}{2}\right)^2 + R^2\right)^{-3/2} + \left(\left(z+\frac{d}{2}\right)^2 + R^2\right)^{-3/2}\right]$$

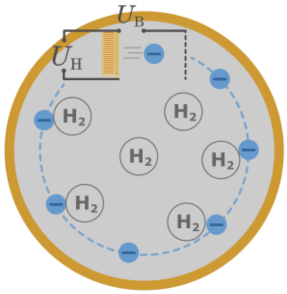
- **Magnetic field $B_{\uparrow\uparrow}(z)$** [T] of a Helmholtz coil along the symmetry axis (z-axis) with the *same* current direction.
- **Magnetic field $B_{\downarrow\uparrow}(z)$** [T] of a Helmholtz coil along the symmetry axis (z-axis) with *opposite* current direction.



Here I [A] is the **magnitude of the current** in each of the coils, R [m] is the **radius** of a coil, N [−] is the **winding number** and d [m] is the **distance** between the two coils. The coordinate z [m] indicates the **location** where the magnetic field is to be calculated.

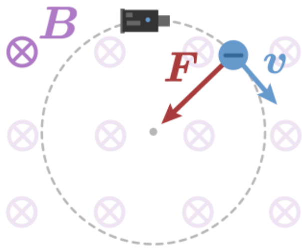
5.8 Teltron tube

$$\frac{q}{m} = \frac{v}{rB} \qquad \frac{q}{m} = \frac{2U_\mathrm{B}}{r^2 B^2}$$



The **specific charge** q/m [C/kg] of a particle of **charge** q [C] and **mass** m [kg] can be determined in a *teltron tube*. For this, the **velocity** v [m/s] of the electrons or the **accelerating voltage** U_B [V] of the electron gun must be given. In addition, the **radius** r [m] and the **magnetic field** B [T], in which the electrons move, must be known.

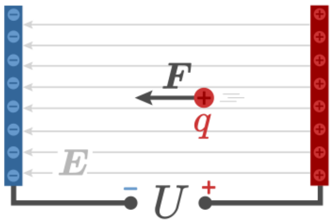
$$v = \frac{2U_\mathrm{B}}{rB}$$



The **velocity** v of the electrons coming out of the electron gun of the teltron tube can be varied with the **accelerating voltage** U_B.

5.9 Parallel plate capacitor

$$E = \frac{U}{d}$$

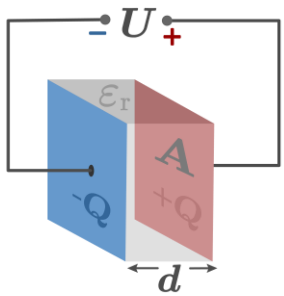


The **electric field** E [V/m] between the plates of a capacitor is homogeneous and depends on the **voltage** U [V] between the plates (electrodes) and their **distance** d [m] from each other.

$$\varphi(x) = -\frac{U}{d}x + \varphi_0$$

Electrical potential $\varphi(x)$ [J/C] between the two capacitor plates (electrodes) increases *linearly* from the first plate to the second plate with the **distance** x [m] and depends on the **distance** d [m] of the plates and the **voltage** U [V] between them. The **potential at the first electrode** is φ_0 [J/C].

$$F = q\frac{U}{d} \quad F = qE$$



The **electric force** F [N] on a **charge** q [C] in a plate capacitor depends on the **electric field** $E = U/d$ [V/m] inside the plate capacitor. The electric field here is **voltage** U [V] per **distance** d [m] of the electrodes.

$$C = \varepsilon_0 \varepsilon_{\mathrm{r}} \frac{A}{d}$$

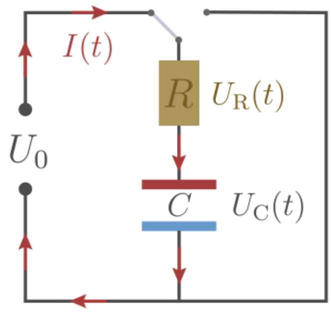
Capacitance C [F] tells how well a plate capacitor can store charge on the electrodes. This storage capacitance depends on the **distance** d [m] between the electrodes and on the inside **inner surface area** A [m^2] of one electrode.

$$F_{\mathrm{e}} = \frac{Q^2}{2\varepsilon_0 \varepsilon_{\mathrm{r}} A} \quad F_{\mathrm{e}} = \frac{\varepsilon_0 \varepsilon_{\mathrm{r}} A}{2d^2} U^2$$

Electric force F_{e} [N] exerted by one capacitor plate with **charge** Q [C] on the other plate. The two plates are at a **distance** d [m] from each other, have an **inner surface area** A [m^2] and a **voltage** U [V] is applied between them.

5.10 Charge and discharge of a capacitor

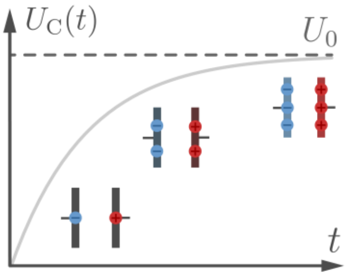
$$I(t) = I_0 \mathrm{e}^{-\frac{t}{RC}}$$



A capacitor with the **capacitance** C [F] is *charged* via a resistor of **resistance** R [Ω] connected in series. The **charging current** $I(t)$ [A] has the **maximum value** I_0 [A] at the beginning of the charging process. The charging current decreases *exponentially* with **time** t [s].

$$U_{\mathrm{R}}(t) = U_0 \mathrm{e}^{-\frac{t}{RC}}$$

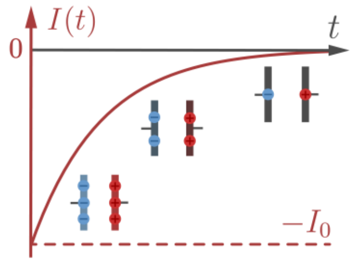
The **voltage** $U_{\mathrm{R}}(t)$ [V] at the resistor with **resistance** R [Ω] also decreases *exponentially* with **time** t from the **initial maximum value** U_0 [A].



$$U_{\mathrm{C}}(t) = U_0\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right)$$

The **capacitor voltage** $U_{\mathrm{C}}(t)$ increases with **time** t [s] during the charging process and reaches the **maximum value** U_0 [V].

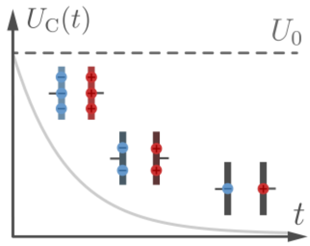
$$I(t) = -I_0\mathrm{e}^{-\frac{t}{RC}} \quad U_{\mathrm{R}}(t) = -U_0\mathrm{e}^{-\frac{t}{RC}}$$



$$U_{\mathrm{C}}(t) = U_0\mathrm{e}^{-\frac{t}{RC}}$$

A capacitor with the **capacitance** C [F] is discharged via a resistor of **reistance** R [Ω] connected in series. The **discharge current** $I(t)$ [A] has the value $-I_0$ [A] at the beginning of the discharging process. With **time** t, the discharge current decreases exponentially to zero. The **voltage** $U_{\mathrm{R}}(t)$ [V] across the resistor also decreases exponentially as the capacitor discharges. The **capacitor voltage** $U_{\mathrm{C}}(t)$ [V] across the capacitor also decreases exponentially from U_0 [V] to zero, but with an opposite sign. Here e is the **Euler number**.

$$\tau = RC \qquad t_{\mathrm{h}} = RC\ln(2)$$

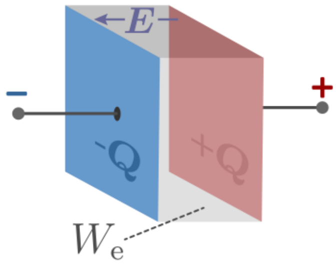


The **time constant** τ[s] characterizes a **R**esistor-**C**apacitor circuit (**RC circuit**) and specifies the time after which the voltage, charge or current at the capacitor has decreased or increased by a *factor of* 1/e.

The **half-life** t_{h} [s] is the time after which an initial value (for example, the voltage) has decreased or increased *by half*. You can use the formula to calculate the half-life for charge, current or voltage on the capacitor.

5.11 Energy (density) of a capacitor

$$W_e = \frac{1}{2}\frac{Q^2}{C} \quad W_e = \frac{1}{2}CU^2$$



$$W_e = \frac{1}{2}\varepsilon_0\varepsilon_r V E^2$$

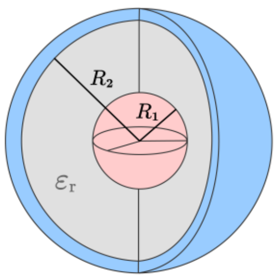
Electrical energy W_e [J] of a capacitor can be calculated using the separated **charge** Q [C] on the electrodes, using the **voltage** U [V] or using the **electric field** E [V/m] between the electrodes. Here V [m^3] is the **volume** enclosed by the two electrodes.

$$w_e = \frac{1}{2}ED \quad w_e = \frac{1}{2}\frac{D^2}{\varepsilon_0\varepsilon_r} \quad w_e = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$$

Electric energy density w_e [J/m^3] indicates the energy of **electric field** E [V/m] or **electric flux density** D [C/m^2] per volume.

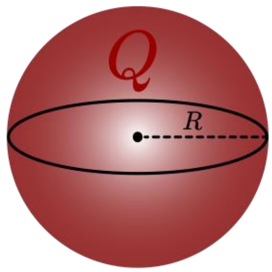
5.12 Capacitance of spheres and cylinders

$$C = 4\pi\varepsilon_0\varepsilon_r\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}$$



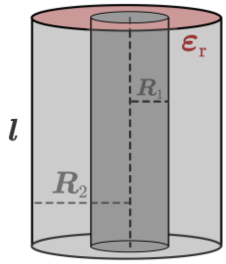
Capacitance C [F] of a *spherical capacitor*. The capacitor consists of an inner sphere electrode with **radius** R_1 [m]. It is surrounded by a dielectric medium with **relative permittivity** ε_r [−]. The spherical outer electrode has **radius** R_2 [m] and encloses the dielectric medium.

$$C = 4\pi\varepsilon_0 R$$



Capacitance C [F] of a *sphere* of **radius** R [m] in a dielectric medium with **relative permittivity** ε_r [−].

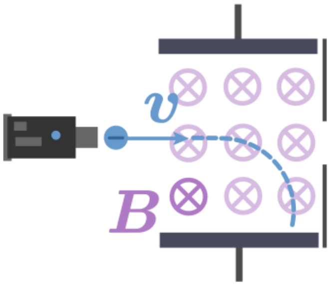
$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\ln\left(\frac{R_2}{R_1}\right)}$$



Capacitance C [F] of a *cylindrical capacitor* (for example of a coaxial cable). The capacitor consists of an inner cylinder with **radius** R_1 [m]. It is surrounded by a dielectric medium with **relative permittivity** ε_r [$-$]. The cylindrical outer electrode has **radius** R_2 [m] and encloses the dielectric.

5.13 Velocity filter (WIEN filter)

$$v = \frac{1}{d}\frac{U}{B} \qquad \Delta v = \frac{mb}{qL^2d^2}\frac{U^2}{B^3}$$

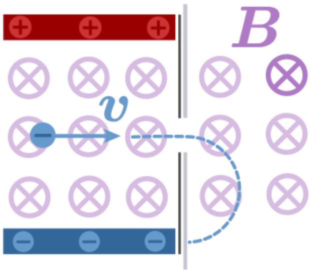


A WIEN filter is a **velocity filter** made of a plate capacitor located in an external **magnetic field** B [T]. At the end of the capacitor is an **aperture of width** b [m]. This setup is bombarded with electrically charged particles. The **length of a capacitor plate** is L [m] and d [m] is the **distance between the plates**. A **voltage** U [V] is applied between the plates. Behind the aperture, charged particles emerge with velocity v [m/s]. A particle has **mass** m [kg] and **charge** q [C]. Since the hole of the aperture has a finite width, in a real WIEN filter particles emerge in a velocity interval.

- The **minimum velocity** of the exiting particles is $v - \Delta v$.
- The **maximum velocity** of the exiting particles is $v + \Delta v$.

5.14 Mass spectrometer

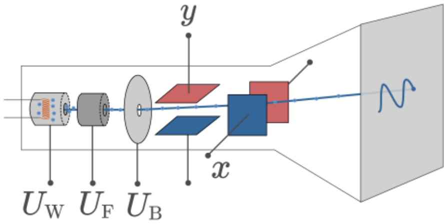
$$m = \frac{qrdB^2}{U}$$



A **mass spectrometer** is a WIEN filter with an additional magnetic field behind the aperture. With this setup, the **mass** m [kg] of a particle of **charge** q [C] can be determined. The plate capacitor is in a **magnetic field** B [T] and the **plates are spaced** d [m] **apart**. On the back side of the aperture, the charged particles land at a distance $2r$ from the opening of the aperture. Thus r [m] is the **radius** of the half-shaped circular path which is created behind the aperture.

5.15 Oscilloscope and Braun tube

$$v_0 = \sqrt{\frac{2eU_B}{m_e}}$$



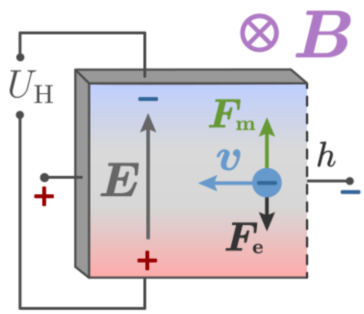
The **initial velocity** v_0 [m/s] of the electrons after they have passed through the **accelerating voltage** U_B [V]. An *electron* carries the **elementary charge** e [C] and has the **mass** m_e [kg].

$$a_y = \frac{eU_y}{m_e d} \quad v_y(t) = \frac{eU_y}{m_e d} t \quad y(t) = \frac{1}{2}\frac{eU_y}{m_e d} t^2$$

The constant **acceleration** a [m/s^2] of the electrons in y-direction depends on the applied **voltage** U_y [V] between the two electrodes and on their **distance** d [m]. The **velocity** $v_y(t)$ [m/s] at time t of the electrons in y-direction is obtained if you multiply the acceleration a by the **time** t [s]. And $y(t)$ [m] is the **distance** covered in y-direction after time t.

5.16 Hall effect

$$U_H = A_H \frac{IB}{d} \quad A_H = \frac{1}{nq}$$

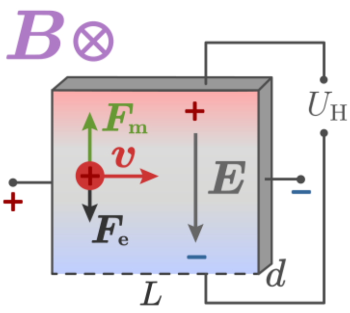


The **Hall voltage** U_H [V] is formed in a Hall bar of **thickness** d [m], which is placed in a perpendicular **external magnetic field** B [T] and through which an **electric current** I [A] flows. The **Hall constant** A_H [m^3/C] depends on the **charge carrier density** n [1/m^3] and the **charge** q [C] of the particle which makes up the current.

$$U_H = vBh$$

Hall voltage can also be calculated with the **drift velocity** v [m/s] of the charge carriers, the **external magnetic field** B and the **height** h [m] of the Hall bar.

$$A_{\mathrm{H}} = \frac{n_{\mathrm{h}}\mu_{\mathrm{h}}^2 - n_{\mathrm{e}}\mu_{\mathrm{e}}^2}{e(n_{\mathrm{h}}\mu_{\mathrm{h}} + n_{\mathrm{e}}\mu_{\mathrm{e}})}$$



- If **electrons** generate the current ($n = n_{\mathrm{e}}$), then the Hall constant is *negative*.
- If **holes** generate the current ($n = n_{\mathrm{h}}$), then the Hall constant is *positive*.
- If *both* **electrons** *and* **holes** contribute to the current, the Hall constant is composed of both charge carrier densities n_{h} and n_{e}! Here μ_{h} $[\mathrm{m^2/Vs}]$ is the **charge carrier mobility of the holes** and μ_{e} $[\mathrm{m^2/Vs}]$ the **charge carrier mobility of the electrons**.

Material		Hall constant A_{H} in $\mathrm{m^3/C}$
Indium	**Hole conduction**	$160 \cdot 10^{-12}$
Aluminium	**Hole conduction**	$99 \cdot 10^{-12}$
Zinkium	**Hole conduction**	$64 \cdot 10^{-12}$
Lithium	**Electron conduction**	$-170 \cdot 10^{-12}$
Natrium	**Electron conduction**	$-248 \cdot 10^{-12}$
Rubidium	**Electron conduction**	$-500 \cdot 10^{-12}$

5.17 Nernst effect (thermal Hall effect)

$$E_y = C_{\mathrm{N}} \frac{T_2 - T_1}{x_2 - x_1} B_z$$

A Hall bar has a (linear) **temperature difference** $T_2 - T_1$ $[\mathrm{K}]$ between the points x_2 $[\mathrm{m}]$ and x_1 $[\mathrm{m}]$ along the x-axis. This causes the electrons to move toward the hot side oft he Hall bar. As this motion happens in a **magnetic field** B_z $[\mathrm{T}]$ (along the z-axis), the electrons are deflected by the Lorentz force, so that an **electric field** E_y $[\mathrm{V}/m]$ is formed along the y-axis. Here, the **Nernst coefficient** C_{N} $[\mathrm{m^2/s} \cdot K]$ is a material-specific quantity that determines how well the electric field can form in the Hall bar.

5.18 Ettingshausen effect

$$\Delta T = C_{\mathrm{E}}\,(x_2 - x_1)\, j_y\, B_z$$

The (linear) **temperature difference** $\Delta T = T_2 - T_1$ $[\mathrm{K}]$ between the points x_2 $[\mathrm{m}]$ and x_1 $[\mathrm{m}]$ of a Hall bar along the x-axis. The Hall bar is in a perpendicular **magnetic field** B_z $[\mathrm{T}]$ (along the z-axis). The temperature difference in the Hall bar is due to an electric current along the y-axis. The electric current is described here by the **electric current density** j_y $[\mathrm{A}/m^2]$. The **Ettingshausen coefficient** C_E $[K \cdot \mathrm{m}^3/\mathrm{J}]$ is a material-specific quantity that determines how well the temperature gradient can form in the platelet.

5.19 Law of mass action

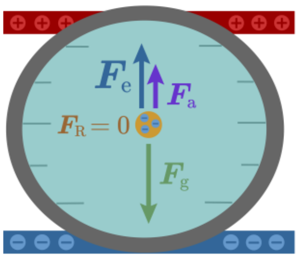
$$n_\mathrm{i} = \sqrt{n_0 p_0}$$

The *law of mass action* describes the (**intrinsic**) **charge carrier density** n_i $[1/\mathrm{m}^3]$ of undoped and doped semiconductors in thermal equilibrium (that is at a *constant* temperature). Here n_0 $[1/\mathrm{m}^3]$ is the **electron density** and p_0 $[1/\mathrm{m}^3]$ the **hole density**.

5.20 Millikan (oil droplet) experiment

$$r = \sqrt{\frac{9\eta v_\downarrow}{2g(\rho_\mathrm{o} - \rho_\mathrm{L})}} \qquad r = \frac{qU}{3\pi d\eta(v_\uparrow + v_\downarrow)}$$

Radius r $[\mathrm{m}]$ of a charged oil **droplet of density** ρ_o $[\mathrm{kg}/\mathrm{m}^3]$ and with **charge** q $[\mathrm{C}]$, which is in a **liquid of density** ρ_L $[\mathrm{kg}/\mathrm{m}^3]$ and **viscosity** η $[\mathrm{Ns}/\mathrm{m}^2]$. The oil droplet is located between two capacitor plates across which a **voltage** U $[\mathrm{V}]$ is applied. Here $v_\downarrow$ $[\mathrm{m}/\mathrm{s}]$ is the **falling velocity** and $v_\uparrow$ $[\mathrm{m}/\mathrm{s}]$ the **rising velocity**.

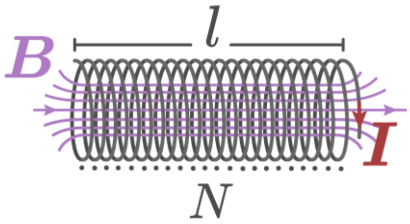


$$q = \frac{9\pi d}{U}\sqrt{\frac{2\eta^3 {v_\downarrow}^3}{g(\rho_\mathrm{o} - \rho_\mathrm{L})}}$$

$$q = \frac{9\pi d}{2U}\sqrt{\frac{\eta^3(v_\downarrow - v_\uparrow)}{g(\rho_\mathrm{o} - \rho_\mathrm{L})}}(v_\uparrow + v_\downarrow)$$

The **charge** q [C] of the oil droplet can be determined either by the *levitation method* (first formula) or by the *uniform field method* (second formula).

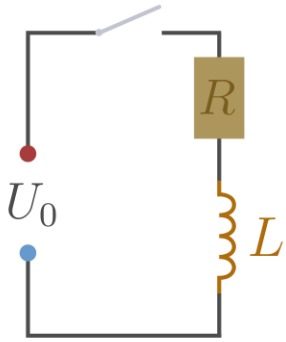
5.21 Coil



$$B = \mu_0 \mu_r \frac{IN}{l}$$

The **magnetic field** B [T] of a coil of **length** l [m] with **winding number** N [−] and a **current** I [A].

$$L = \mu_0 \mu_r \frac{AN^2}{l}$$

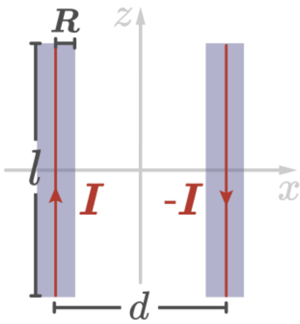


The **inductance** L [H] of the coil depends on the **cross-sectional area** A [m^2], on the **winding number** N [−] and on the **length** l [m] of the coil.

$$I(t) = I_{\text{max}} \left(1 - e^{-\frac{R}{L}t}\right)$$

Electric current $I(t)$ [A] at **time** t [s] through a coil with **inductance** L [H] and with a resistor of **resistance** R [Ω] connected in series. Here $I_{\text{max}} = U_0/R$ [A] is the **maximum current** which arises after the magnetic field of the coil has built up. And U_0 [V] is the applied **source voltage**.

5.22 (Self)inductance of two current-carrying wires

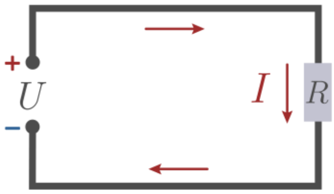


$$L = \frac{\mu_0 l}{\pi}\left[\frac{1}{2} + \ln\left(\frac{d-R}{R}\right)\right]$$

Inductance L [H] of two round wires with **radius** R [m] and **length** l [m], which are located at a **distance** d [m] from each other and whose currents flow in opposite directions. The wires are *inductively* coupled. Here μ_0 is the **vacuum permeability**.

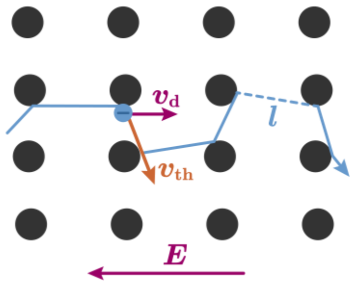
5.23 Ohm's law, current density and conductance

$$U = RI$$



Ohm's law states that the **voltage** U [V] is linearly related to the **current** I [A]. The constant of proportionality is called **resistance** R [Ω]. Ohm's law applies to most metallic conductors.

$$j = \sigma E \quad j = \frac{ne^2\tau}{m_e}E \quad j = nev_d$$



In an *isotropic* (direction-independent) material, the **current density** j [A/m²] is proportional to the applied **electric field** E [V/m]. The constant of proportionality is the **electrical conductivity** σ [1/Ωm]. According to the *Drude model*, which describes a classical charge transport, the conductivity depends on the material-dependent **mean free time** (**impact time**) τ [s] and the **charge carrier density** n [1/m³]. Here e is the **elementary charge** and m_e [kg] the **electron mass**. The current density can also be written with the **drift velocity** v_d [m/s], which describes a *directed* movement of the electrons due to the applied electric field.

$$G = \frac{1}{R}$$

Conductance G [S = 1/Ω] (S stands for *Siemens*) is the reciprocal of the **resistance** R [Ω].

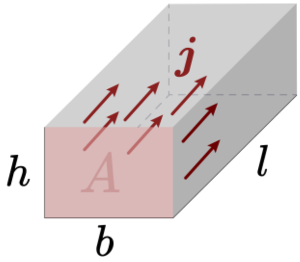
5.24 Wiedemann-Franz law

$$\frac{\kappa}{\sigma} = LT$$

Specific thermal conductivity κ [W/mK] tells how well a conductor can transport *heat*, while **specific electrical conductivity** σ [1/Ωm] tells how well a conductor conducts *electric current*. The *Wiedemann-Franz law* states that the ratio of the two conductivities is proportional to the **temperature** T [K] of the conductor under consideration. The constant of proportionality is the **Lorenz number** $L \approx 2.44 \cdot 10^{-8}$ [W · Ω/K²]. The Wiedemann-Franz law is well satisfied in metals at very low and very high temperatures (compared to the *Debye temperature*).

5.25 (Specific) resistance of a wire

$$R = \frac{l}{A}\rho$$



Resistance R $[\Omega]$ depends on the geometry of the wire, that is its **length** l $[\mathrm{m}]$ and **cross-sectional area** A $[\mathrm{m}^2]$. Here ρ $[\Omega\mathrm{m}]$ is the **specific electrical resistance**, which depends on the temperature and the material considered.

Material		Specific resistance ρ in $\Omega\mathrm{m}$	Specific resistance ρ in $\Omega \cdot \mathrm{mm}^2/\mathrm{m}$
Aluminium	Conductor	$2.65 \cdot 10^{-8}$	0.0265
Plumbium (Lead)	Conductor	$2.08 \cdot 10^{-7}$	0.208
Blood	Hole conductor	1.6	$\approx 1.6 \cdot 10^{6}$
Glass	Insulator	10^{10} to 10^{15}	10^{16} to 10^{21}
Cuprium (Copper)	Conductor	$1.7 \cdot 10^{-8}$	0.017
Seawater	Conductor	0.5	$5 \cdot 10^{5}$
Stannium (Tin)	Conductor	$1.09 \cdot 10^{-7}$	0.109
Nickelium (Nickel)	Conductor	$6.93 \cdot 10^{-8}$	0.0693

Table 5.1: Specific resistance at 20 °C. To convert the typical unit of spec. resistance $\Omega \cdot mm^2/m$ to Ωm, multiply the value in $\Omega \cdot mm^2/m$ by 10^{-6}.

$$\rho(T) = \rho_0(1 + \alpha(T - T_0) + \beta(T - T_0)^2)$$

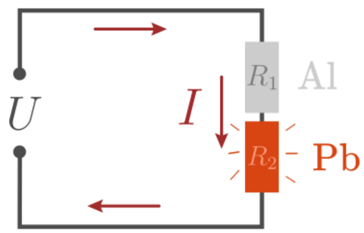
Specific electrical resistance $\rho(T)$ $[\Omega\mathrm{m}]$ of a metal as a function of **temperature** T $[\mathrm{K}]$. Here, ϱ_0 is the **specific resistance at the reference temperature** T_0 $[\mathrm{K}]$ (for example at room temperature). The **temperature coefficients** α $[1/\mathrm{K}]$ and β $[1/\mathrm{K}^2]$ depend on the material. In many cases, the temperature dependency can be simplified, meaning $\beta = 0$.

Material		Temperature coefficient α in $1/\mathrm{K}$
Aluminium	Conductor	$3.9 \cdot 10^{-3}$
Plumbium (Lead)	Conductor	$4.2 \cdot 10^{-7}$
Cuprium (Copper)	Conductor	$3.9 \cdot 10^{-3}$
Stannium (Tin)	Conductor	$4.5 \cdot 10^{-3}$
Nickelium (Nickel)	Conductor	$6.7 \cdot 10^{-3}$

5.26 Electric power and work

$$P = \frac{\Delta W}{\Delta t} \quad P = UI \quad P = \frac{U^2}{R}$$

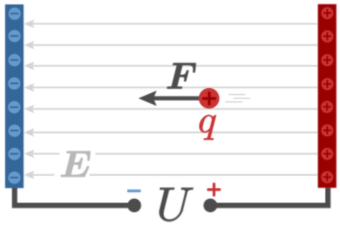
$$P = RI^2$$

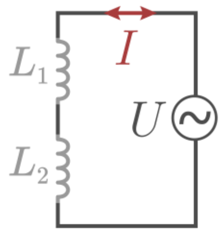


Power P [W = J/s] is **work** done ΔW [J] (energy *released* or *consumed*) per time Δt [s]. In the case of electrical circuits, *electric* power can be expressed in terms of voltage and current. **Voltage** U [V] can be applied to resistor of **resistance** R [Ω] through which **current** I [A] flows.

$$W = q \cdot U \quad W = U \cdot I \cdot t$$

$$W = P \cdot t$$

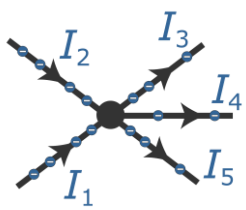




Work W [J] done on/by a **charge** q [C] when it passed through **voltage** U [V]. The transported charge can also be expressed by the **current** I [A], which has passed within the **time** t [s]. The product UI corresponds to the **electric power** P [W].

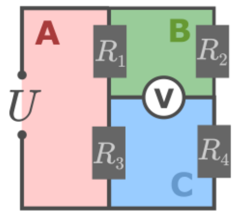
5.27 Kirchhoff's circuit laws

$$I_1 + I_2 + I_3 + \ldots + I_n = 0$$



The *Kirchhoff's current law* states that the sum of the **currents** I_1, I_2 to I_n [A] flowing into or out of a node of a circuit is equal to zero.

$$U_1 + U_2 + U_3 + \ldots + U_n = 0$$



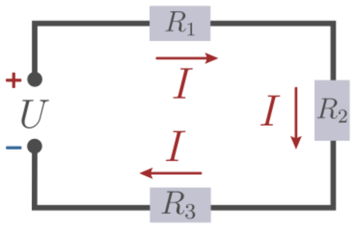
The *Kirchhoff's voltage law* states that the sum of the **voltages** U_1, U_2 to U_n [V] of a closed circuit loop is equal to zero.

5.28 Series circuits

$$R = R_1 + R_2 + R_3 + \dots + R_n$$

$$I = I_1 = I_2 = I_3 = \dots = I_n$$

$$U = U_1 + U_2 + U_3 + \dots + U_n$$



In the case of a *series connection of resistors*, the individual **resistances** R_1, R_2, R_3, ... [Ω] add up to a **total resistance** (**equivalent resistance**) R [Ω]. The **currents** I_1, I_2, I_3, ... [A] through resistors are all equal to the **total current** I [A] in the main line. And the sum of all **voltages** U_1, U_2, U_3, ... [V] at the resistors corresponds to the **applied voltage** U [V].

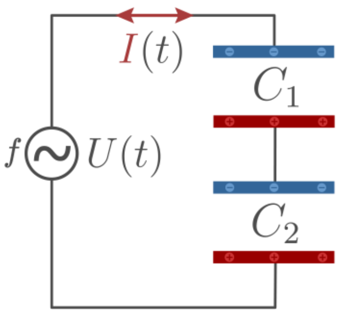
$$L = L_1 + L_2 + L_3 + \dots + L_n$$

In a *series connection of coils*, the **individual inductances** L_1, L_2, L_3, ... [H] and so on add up to a **total inductance** (**equivalent inductance**) L [H].

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

$$Q = Q_1 = Q_2 = Q_3 = \dots = Q_n$$

$$U = U_1 + U_2 + U_3 + \dots + U_n$$



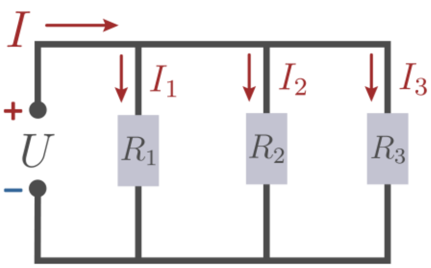
In a *series circuit of capacitors*, the individual **capacitances** C_1, C_2, C_3, ... [F] add up reciprocally to a reciprocal **total capacitance** (**equivalent capacitance**) C [F]. The **charges** Q_1, Q_2, Q_3, ... [C] are equal to the **total charge** Q [C] on the capacitors. And the **voltages** U_1, U_2, U_3, ... [V] on the capacitors in the sum form the **applied voltage** U [V].

5.29 Parallel circuits

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

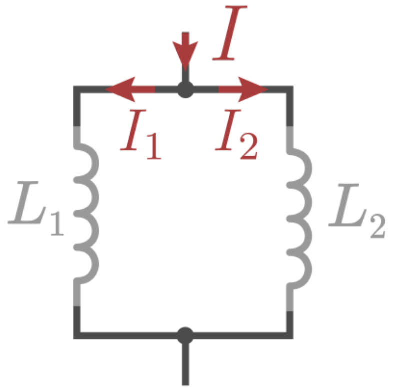
$$I = I_1 + I_2 + I_3 + \dots + I_n$$

$$U = U_1 = U_2 = U_3 = \dots = U_n$$



In the case of a *parallel connection of resistors*, the **individual resistances** R_1, R_2, R_3, ... [Ω] add up reciprocally to a reciprocal **total resistance** (**equivalent resistance**) R [Ω]. So after you have substituted the resistance values into equation, you have to form the reciprocal of the result to get R. The **currents** I_1, I_2, I_3, ... [A] add up to the **total current** I [A] in the main line. And the **voltages** U_1, U_2, U_3, ... [V] at the resistors are equal to the **applied voltage** U [V].

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}$$

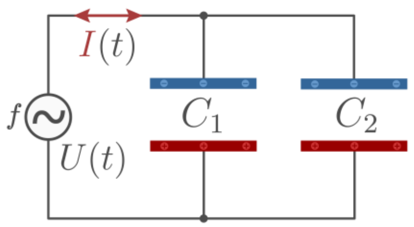


In the case of a *parallel connection of coils*, the individual **inductances** L_1, L_2, L_3, ... [H] add up reciprocally to a reciprocal **total inductance** (**equivalent inductance**) L [H].

$$C = C_1 + C_2 + C_3 + \ldots + C_n$$

$$Q = Q_1 + Q_2 + Q_3 + \ldots + Q_n$$

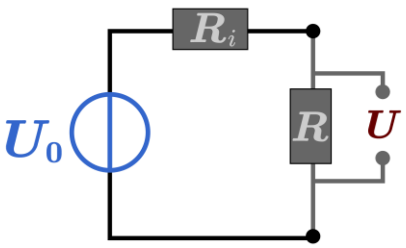
$$U = U_1 = U_2 = U_3 = \ldots = U_n$$



In the case of a *parallel connection of capacitors*, the individual **capacitances** C_1, C_2, C_3, ... [F] are added up to a **total capacitance** (**equivalent capacitance**) C [F]. The **charges** Q_1, Q_2, Q_3, ... [C] add up to a **total charge** Q [C], which is stored in all capacitors. And the individual **voltages** U_1, U_2, U_3, ... [V] at the capacitors are equal to the **applied voltage** U [V].

5.30 Real-world voltage source

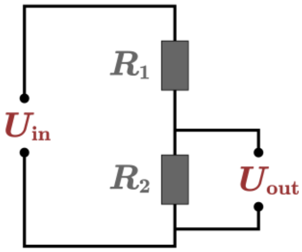
$$U = \frac{U_0}{1 + \frac{R_\mathrm{i}}{R}}$$



Terminal voltage U [V], that is the actual voltage at the **load resistor with resistance** R [Ω]. It depends on the **internal resistance** R_i [Ω] of a *real* voltage source. If the internal resistance R_i was not there, then the terminal voltage would be equal to the **source voltage** U_0 [V]. In this case, we would have an *ideal* voltage source.

5.31 Voltage divider

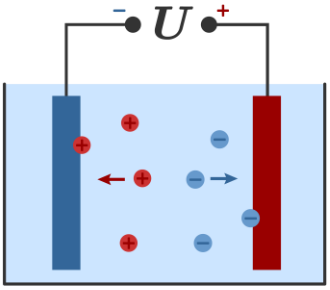
$$U_{\text{out}} = \frac{R_2}{R_1 + R_2} U_{\text{in}}$$



Output voltage U_{out} [V] across the **second resistor with resistance** R_2 [Ω]. It depends on the selected **first series resistor with resistance** R_1 [Ω] and on the applied **input voltage** U_{in} [V].

5.32 Electrolysis

$$\Delta m = \frac{M}{Ze} I \Delta t \quad F = N_{\text{A}} e \quad A_{\text{e}} = \frac{M}{ZF}$$

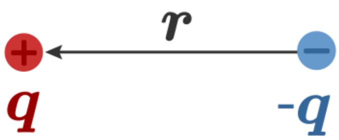


Faraday's law describes the **mass** Δm [kg] of all ions deposited on an electrode within the **time** Δt [s]. The mass Δm is therefore the difference between the mass of the electrode before and after electrolysis. During electrolysis, a **current** I [A] is generated between the electrodes. Here, M [kg] is the **mass of a single ion** deposited on an electrode (for example, mass of an Na^+ ion). The **charge** Ze [C] of an ion is a multiple of the **elementary charge** e [C], with $Z = 1,2,3, \ldots$

In electrolysis, the **electrochemical equivalent** A_{e} [g/C] (grams per coulomb) indicates how much mass of a substance is deposited when 1 coulomb of charge is transported from one electrode to another. Here $F = 9.648 \cdot 10^4$ C/mol is the **Faraday constant**.

5.33 Electric dipole

$$d = qr \quad d = \alpha E$$



Electric dipole moment d [Cm] of two oppositely **charges of magnitude** q [C], which are located at a **distance** r [m] from each other. The dipole moment, for example of a molecule, can be generated by an **external electric field** E [V/m]. Here α $[\text{Cm}^2/\text{V}]$ is the **polarizability**, which describes how easy it is to generate a dipole.

Atom/Molecule	**Polarizability α in Cm^2/V**
Helium (He)	$23 \cdot 10^{-42}$
Hydrogenium (H)	$74 \cdot 10^{-42}$
Water (H_2O)	$167 \cdot 10^{-42}$

Atom/Molecule	Polarizability α in $\mathrm{Cm^2/V}$
Molecular oxygen (O_2)	$173 \cdot 10^{-42}$
Carbon dioxide (CO_2)	$279 \cdot 10^{-42}$
Natrium (Na)	$2681 \cdot 10^{-42}$

$$M = E d \sin(\varphi)$$

Torque M [Nm] on a dipole in the homogeneous **electric field** E [V/m], if the dipole is at an **angle** φ [rad] to electric field lines. The **dipole moment** d [Cm] and the electric field lie in one plane and the torque is *perpendicular* to this plane.

$$E_\mathrm{d}(x) = \frac{d}{4\pi\varepsilon_0\varepsilon_\mathrm{r}}\frac{1}{x^3} \quad E_\mathrm{d}(y) = \frac{d}{4\pi\varepsilon_0\varepsilon_\mathrm{r}}\frac{1}{y^3} \quad x, y \gg r$$

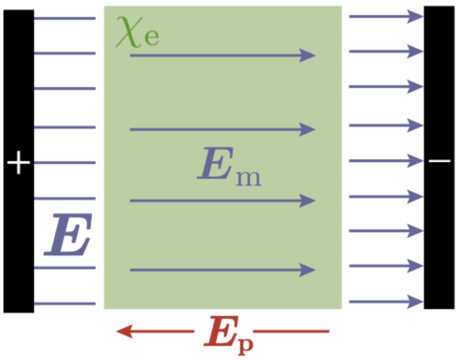
An electric dipole with **dipole moment** $d = qr$, is aligned parallel to the y-axis. The magnitude of the **electric field** E_d [V/m], on the *parallel* y-axis and on the x-axis *perpendicular* to it, decreases cubically with the **distance** x [m] and y [m], respectively. The formulas are valid only for distances x, y, which are much larger than the **length** r [m] of the dipole.

5.34 Electric susceptibility and polarization

$$P = \chi_\mathrm{e}\varepsilon_0 E \qquad E_\mathrm{p} = \frac{P}{\varepsilon_0}$$

$$E_\mathrm{m} = \frac{E}{1 + \chi_\mathrm{e}} \qquad E_\mathrm{m} = E - \frac{P}{\varepsilon_0}$$

$$\chi_\mathrm{e} = \frac{N\alpha}{\varepsilon_0 V} \qquad \chi_\mathrm{e} = \varepsilon_\mathrm{r} - 1$$



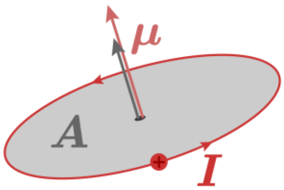
Electric susceptibility χ_e [−] indicates how well a material can be polarized by an **external electric field** E [V/m]. Here ε_r [−] is the **relative permittivity**. **Dielectric polarization** P [$\mathrm{C/m^2}$] describes the density of electric dipoles in a material and induces a **polarization field** E_p [V/m], which points opposite or in the direction of the external field and thus amplifies or weakens the external E-field.

The external E-field becomes the **damped** ($\chi_\mathrm{e} > 0$) or **amplified** ($\chi_\mathrm{e} < 0$) **electric field** E_m [V/m] **inside the material**. The case $\chi_\mathrm{e} = 0$ corresponds to vacuum. Susceptibility can be expressed with microscopic quantities, namely **polarizability** α $[\mathrm{Cm^2/V}]$, **number** N $[-]$ **of dipoles** per **volume** V $[\mathrm{m^3}]$.

Medium	Electric susceptibility χ_e
Vacuum	0
Air (0°C)	0.0005
Glass	4 to 9
Water (0°C)	87
Water (40°C)	72.4
Ice (-20°C)	15
Hydrogen cyanide	94
Ethanol (20°C)	24.8

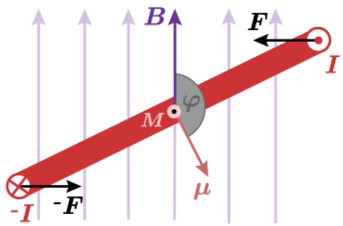
5.35 Magnetic dipole

$$\mu = IA$$



Magnetic dipole moment μ $[\mathrm{Am^2}]$ generated by a circular **current** I $[\mathrm{A}]$ enclosing an **area** A $[\mathrm{m^2}]$.

$$M = \mu B \sin(\varphi)$$



Torque M $[\mathrm{Nm}]$ experienced by a dipole with **dipole moment** μ $[\mathrm{Am^2}]$ in a homogeneous **magnetic field** B $[\mathrm{T}]$. Here φ $[\mathrm{rad}]$ is the **angle** between the magnetic field lines and the dipole moment vector. The formula simplifies to $M = \mu B$ when the dipole is aligned *parallel* to the field lines.

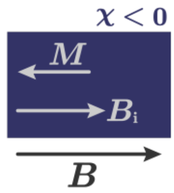
$$W_\mu = -\mu B \cos(\varphi)$$

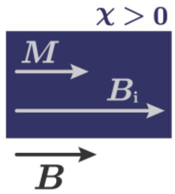
Potential energy W_μ $[\mathrm{J}]$ of a dipole with **magnetic dipole moment** μ $[\mathrm{Am^2}]$ in **magnetic field** B $[\mathrm{T}]$.

5.36 Magnetic susceptibility and magnetization

$$\chi_m = \mu_r - 1 \qquad \tilde{\chi}_m = \frac{\tilde{m}}{\rho}\chi_m$$

Magnetic susceptibility χ_{m} $[-]$ and **molar magnetic susceptibility** $\tilde{\chi}_{\mathrm{m}}$ $[\mathrm{m^3/mol}]$ indicate how well a material can be magnetized by an external magnetic field B. Here μ_{r} $[-]$ is the **relative permeability**, $\tilde{m}$ $[\mathrm{kg/mol}]$ is the **molar mass**, and ρ $[\mathrm{kg/m^3}]$ is the **mass density** of the material.





- If the magnetic susceptibility is negative: $-1 < \chi_{\mathrm{m}} < 0$, then the material is **diamagnetic**.
- If the magnetic susceptibility is positive: $\chi_{\mathrm{m}} > 0$, then the material is **paramagnetic**.
- If the magnetic susceptibility is much greater than zero $\chi_{\mathrm{m}} \gg 0$, then the material is **ferromagnetic**.

Material at 20 °C and 1 atm		**Magnetic susceptibility** χ_{m}	**Molar susceptibility** $\tilde{\chi}_{\mathrm{m}}$
Superconductor	diamagnetic	-1	–
Diamond	diamagnetic	$-2.2 \cdot 10^{-5}$	$-7.4 \cdot 10^{-11}$
Cuprium (Cu) „Copper“	diamagnetic	$-9.6 \cdot 10^{-6}$	–
Water (H2O)	diamagnetic	$-9.0 \cdot 10^{-6}$	$-1.6 \cdot 10^{-10}$
Helium (He)	diamagnetic	$-9.9 \cdot 10^{-10}$	$-2.4 \cdot 10^{-11}$
Aluminium (Al)	paramagnetic	$+2.2 \cdot 10^{-5}$	$+2.2 \cdot 10^{-10}$
Nickelium (Ni)	ferromagnetic	600	–
Ferrium (Fe) „Iron“	ferromagnetic	$200\,000$	–

$$M = \chi_{\mathrm{m}} H \qquad M = \frac{\chi_{\mathrm{m}}}{\mu_0 \mu_{\mathrm{r}}} B$$

Magnetization M $[\mathrm{A/m}]$ describes the number of magnetic dipoles per volume of a material. Here H $[\mathrm{A/m}]$ is the **magnetic field strength** and B $[\mathrm{T}]$ the **magnetic flux density**.

5.37 Self-inductance of a straight wire, wire loop and a coil

$$U_{\mathrm{ind}} = -L \frac{\Delta I}{\Delta t}$$

Induced voltage U_{ind} $[\mathrm{V}]$, which arises between the ends of a *wire* when the **current** $\Delta I = I_2 - I_1$ $[\mathrm{A}]$ has changed (linearly) in this wire within the **time** $\Delta t = t_2 - t_1$ $[\mathrm{s}]$. Here L $[\mathrm{H} = \mathrm{Ws/A}]$ is the **inductance** of the wire. The *Lenz rule* is expressed by the minus sign and states that the induced voltage tries to impede the current change.

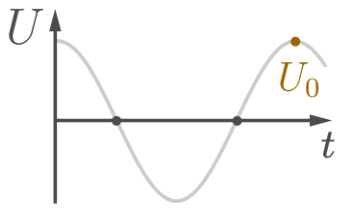
$$U_{\text{ind}} = -\frac{\Delta\Phi}{\Delta t} \quad U_{\text{ind}} = -N\frac{\Delta\Phi}{\Delta t}$$

Induced voltage U_{ind} [V], which arises between the ends of a *wire loop*, if the **magnetic flux** $\Delta\Phi = \Phi_2 - \Phi_1$ [Vs], through the wire loop, has changed (linearly) within the **time** $\Delta t = t_2 - t_1$ [s]. If the wire loop has **winding number** N [−], then it is a *coil* and the induced voltage is then N times as large.

1. Induced voltage $U_{\text{ind}} = -B\frac{\Delta A}{\Delta t}$ due to the *temporal change* ΔA *of the area* penetrated by the **magnetic field** B [T].
2. Induced voltage $U_{\text{ind}} = -A\frac{\Delta B}{\Delta t}$ due to the *temporal change* ΔB *of the magnetic field* penetrating the **area** A [m^2].
3. Induced voltage $U_{\text{ind}}(t) = ANB\omega\sin(\omega t)$ due to *periodic angular change* ωt between the magnetic field and the surface orthonormal vector. Here ω [rad/s] is the constant **angular frequency** at which a wire loop rotates. The **amplitude** of the induced voltage is $ANB\omega$ [V].

5.38 Alternating (AC) voltage

$$U(t) = U_0\cos(\omega t)$$



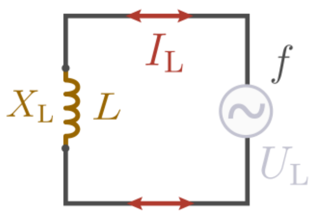
A harmonic **AC voltage** $U(t)$ [V] with the **angular frequency** $\omega = 2\pi f$ [rad/s] and the **peak voltage** (**maximum value**) U_0 [V]. Here f [Hz] is the **frequency** of the voltage.

5.39 AC voltage across a coil (inductance)

$$U_{\text{L}}(t) = U_0\cos(\omega t)$$

$$I_{\text{L}}(t) = I_0\cos(\omega t - \pi/2)$$

$$I_0 = \frac{U_0}{X_{\text{L}}} \quad I_{\text{rms}} = \frac{U_{\text{rms}}}{X_{\text{L}}}$$



AC Voltage $U_{\text{L}}(t)$ [V] at time t [s] applied between the ends of the coil causes a phase-shifted **AC current** $I_{\text{L}}(t)$ [A] with **maximum value** I_0 [A]. Hierbei ist U_0 [V] der **Maximalwert der Spannung**. Both the voltage and the current through the coil oscillate at the **angular frequency** ω [rad/s]. The **rms current** I_{rms} [A] through the coil is the **rms voltage** U_{rms} [V] divided by the **inductive resistance** X_{L} [Ω] of the coil.

$$X_L = 2\pi f L \quad X_L = \omega L$$

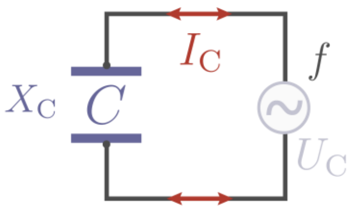
Inductive reactance X_L [Ω] of the coil depends on the **frequency** f [Hz] of the AC voltage and on the **inductance** L [H] of the coil. Here $\omega = 2\pi f$ [rad/s] is the **angular frequency**.

5.40 AC voltage across a capacitor (capacitance)

$$U_C(t) = U_0 \cos(\omega t)$$

$$I_C(t) = I_0 \cos(\omega t + \pi/2)$$

$$I_0 = \frac{U_0}{|X_C|} \quad I_{rms} = \frac{U_{rms}}{|X_C|}$$



AC Voltage $U_C(t)$ [V] at **time** t [s], applied between the electrodes of the capacitor, causes a phase-shifted **AC current** $I_C(t)$ [A] with the **maximum value** I_0 [A]. Here U_0 [V] is the **maximum value of the voltage**. Both the voltage and the current through the coil oscillate at the **angular frequency** ω [rad/s]. The **rms current** I_{rms} [A] through the capacitor is the **rms voltage** U_{rms} [V] divided by the **capacitive resistance** X_C [Ω] of the capacitor.

$$X_C = -\frac{1}{2\pi f C} \quad X_C = -\frac{1}{\omega C}$$

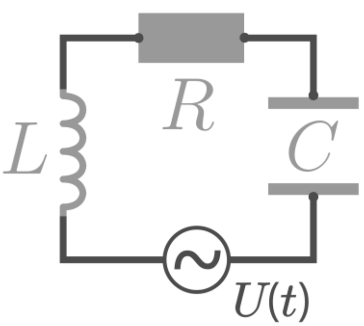
Capacitive reactance X_C [Ω] of a capacitor depends on the **frequency** f [Hz] of the AC voltage and on the **capacitance** C [F] of the capacitor. Here $\omega = 2\pi f$ is the **angular frequency**.

5.41 RLC series circuit (resistor, coil, capacitor)

$$U(t) = U_0 \cos(\omega t) \quad I(t) = I_0 \cos(\omega t - \varphi) \quad \varphi = \arctan\left(\frac{X_L - |X_C|}{R}\right)$$

$$I_0 = \frac{U_0}{Z} = \frac{U_0}{\sqrt{R^2 + (X_L - |X_C|)^2}}$$

$$I_{rms} = \frac{U_{rms}}{Z} = \frac{U_{rms}}{\sqrt{R^2 + (X_L - |X_C|)^2}}$$

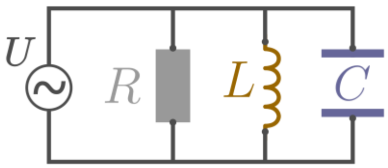


A series RLC circuit consists of an ohmic resistor with **resistance** R [Ω], a coil with **inductive resistance** X_L [Ω] and a capacitor with **capacitive resistance** X_C [Ω]. An **AC voltage** $U(t)$ [V] at **time** t [s] applied to the

RLC series circuit causes an **AC current** $I(t)$ [A] phase-shifted by **angle** φ [rad] with **maximum value** I_0 [A]. Here, U_0 [V] is the **maximum value of the voltage**. The **rms current** I_{rms} [A] through the RLC circuit is the **rms voltage** U_{rms} [V] divided by the **total impedance** Z [Ω] of the circuit.

5.42 RLC parallel circuit (resistor, coil, capacitor)

$$U(t) = U_0 \cos(\omega t) \qquad I(t) = I_0 \cos(\omega t - \varphi)$$



$$I_0 = \frac{U_0}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\, U_0$$

$$I_{\text{rms}} = \frac{U_{\text{rms}}}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\, U_{\text{rms}}$$

A parallel RLC circuit consists of an ohmic resistor with **resistance** R [Ω], a coil with **inductance** L [H] and a capacitor with **capacitance** C [F]. An **AC voltage** $U(t)$ [V] at **time** t [s] applied to the RLC series circuit causes an **AC current** $I(t)$ [A] phase-shifted by **angle** φ [rad] with **maximum value** I_0 [A]. Here, U_0 [V] is the **maximum value** of the voltage (**peak voltage**). Both the voltage and the current through the coil oscillate with the **angular frequency** ω [rad/s]. The **rms current** I_{rms} [A] through the RLC circuit is the **rms voltage** U_{rms} [V] divided by the **total impedance** Z [Ω] of the circuit.

5.43 Power (active, reactive, apparent)

$$P(t) = U_0 I_0 \sin(\omega t) \sin(\omega t - \varphi)$$

$$P_{\text{w}} = U_{\text{rms}} I_{\text{rms}} \cos(\varphi) \qquad P_{\text{w}} = I_{\text{rms}}{}^2 R$$

$$P_{\text{s}} = U_{\text{rms}} I_{\text{rms}} \qquad P_{\text{s}} = \sqrt{P_{\text{s}}{}^2 + P_{\text{b}}{}^2} \qquad P_{\text{s}} = I_{\text{rms}}{}^2 |Z| \qquad P_{\text{s}} = \frac{U_{\text{rms}}{}^2}{|Z|}$$

$$P_{\text{b}} = P_{\text{s}} \sin(\varphi) \qquad P_{\text{b}} = I_{\text{rms}}{}^2 X$$

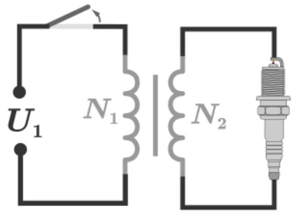
Power $P(t)$ [W] at **time** t [s] oscillates periodically with **angular frequency** ω [rad/s], where current reaches **maximum value** I_0 [A] and voltage reaches **maximum value** U_0 [V]. Here, the **angle** φ [rad] indicates the phase shift between current and voltage. The *usable* power is the **active power** P_{w} [W = J/s], while the **reactive power** P_{b} [var = W] is not usable. The **apparent power**

P_s [VA] is composed of the active and reactive power and is the product of the **rms voltage** U_{rms} [V] and the **rms current** I_{rms} [A]. Here X [Ω] is the **imaginary part** of the **impedance** Z [Ω] (complex total resistance of a circuit).

5.44 Transformer

$$\frac{U_{rms,1}}{U_{rms,2}} = \frac{N_1}{N_2}$$

$$U_{rms,1} I_{rms,1} \cos(\varphi_1) = U_{rms,2} I_{rms,2} \cos(\varphi_2)$$

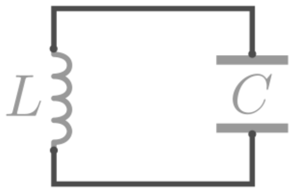


An unloaded transformer with **rms voltage** $U_{rms,1}$ [V] in the *primary* coil and **winding number** N_1 [−] generates **rms voltage** $U_{rms,2}$ [V] in the *secondary* coil which has the **winding number** N_2 [−]. Here $I_{rms,1}$ [A] is the **rms current** through the primary coil and $I_{rms,2}$ [A] through the secondary coil. And φ_1 [rad] is the **phase shift** between the current and voltage in the primary coil. When the transformer is heavily loaded: $\varphi_1 = \varphi_2$.

5.45 Electrical resonance

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \omega_r = \frac{1}{\sqrt{LC}} \qquad T = 2\pi\sqrt{LC}$$

$$I_0 = \frac{Q_0}{\sqrt{LC}}$$



Resonant frequency (**natural frequency**) f_r [Hz], **resonant angular frequency** ω_r [1/s] and **period** T [s] of an *undamped* **LC resonant circuit** with **inductance** L [H] and **capacitance** C [F]. **Peak current** I_0 [A] (maximum current) is the current flowing through the circuit at certain times. Here Q_0 [C] is the **maximum charge** of the capacitor. It corresponds to the charge brought onto the capacitor during charging.

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad T = 2\pi\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{-1/2}$$

$$I(t) = I_0 e^{-\frac{R}{2L}t} \cos(2\pi f_r t)$$

Resonant frequency (**natural frequency**) f_r [Hz], **resonant angular frequency** ω_r [1/s] and **period** T [s] of a *damped* **RLC resonant circuit** with **inductance** L [H], **capacitance** C [F] and

resistance R [Ω]. The **AC current** $I(t)$ [A] through the circuit decreases exponentially from the initial **maximum value** I_0 [A] with **time** t [s].

5.46 Diode

$$I(U) = I_S\left(e^{\frac{U}{nU_T}} - 1\right) \qquad U_T = \frac{k_B T}{e}$$

The *Shockley equation* describes the **diode current** $I(U)$ [A] through a diode as a function of the **diode voltage** U [V] (**forward voltage**). The current depends on the **temperature** T [K], which occurs in the definition of the **temperature voltage** U_T [V]. Here n [−] is the **emission coefficient** (**ideality factor**) and is approximately in the range between 1 and 2. And k_B is the **Boltzmann constant** and e [C] is the **elementary charge**. To determine the temperature-dependent **reverse current** I_S [A], the diode is operated in the reverse direction.

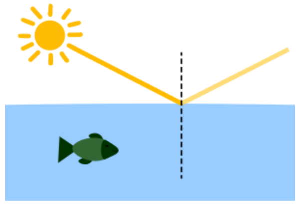
$$d(U) = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(U_d - U)}$$

Width $d(U)$ [m] of the depletion region, which arises at the boundary layer between the p- and n-doped semiconductor due to the **applied voltage** U [V]. Depending on where the positive or negative terminal is, the **pn diode** is *reverse* or *forward* biased. Here, N_A [1/m^3] is the **acceptor concentration** (density of dopant atoms in the p-layer) and N_D [1/m^3] is the **donor concentration** (density of dopant atoms in the n-layer). A **diffusion voltage** U_d [V] is generated in the depletion region. The **relative permittivity** ε_r [−] describes the medium in the depletion region. Here ε_0 [As/Vm] is the **vacuum permittivity** and e [C] the **elementary charge**.

6. OPTICS

Behavior of light and its interaction with matter.

6.1 Reflection, refraction and speed of light



$$\varphi = \varphi_{\mathrm{r}}$$

A light beam incident on a surface at the **angle of incidence** φ [rad] is reflected at the **angle of reflection** φ_{r} [rad]. The two angles are equal.

$$n = \frac{c}{c_{\mathrm{m}}}$$

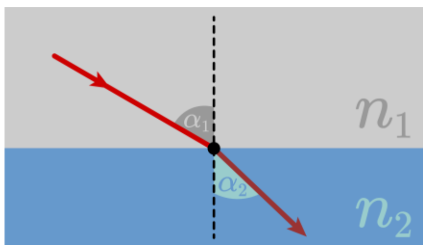
Refractive index n [−] is the ratio of the **speed of light** c [m/s] **in vacuum** to the **speed of light** c_{m} [m/s] **in a medium**.

Medium	Refractive index n	Speed of light in the medium c_{m} in m/s
Vakuum	1	$3 \cdot 10^8$
Air	1	$3 \cdot 10^8$
Water (20 °C)	1.3	$2.3 \cdot 10^8$
Window glass	1.5	$2 \cdot 10^8$
Diamond	2.4	$1.25 \cdot 10^8$

Table 6.1: Approximate values for refractive indices of various media with light wavelength 590 nanometers.

$$\frac{\lambda_1}{\lambda_2} = \frac{c_1}{c_2}$$

When light passes from medium #1 to medium #2, the light **wavelength** changes from λ_1 [m] to λ_2 [m] and the **speed of light** changes from c_1 [m/s] to c_2 [m/s]. The light frequency does not change.



$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

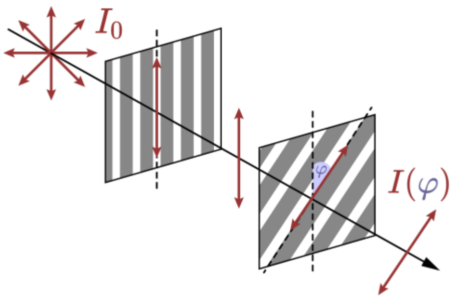
Snell's law describes a light beam falling from a medium with **refractive index** n_1 [−] at an **angle of incidence** α_1 [rad] into a medium having a different **refractive index** n_2 [−] and the light beam has an **angle of incidence** α_2 [rad].

$$\alpha_{\mathrm{c}} = \arcsin\left(\frac{n_2}{n_1}\right)$$

Total **I**nternal **R**eflection (**TIR**) occurs when the light beam enters the medium at the **critical angle** α_c [rad]. The critical angle depends on the **refractive indices** n_1 [−] and n_2 [−] of the first and second medium.

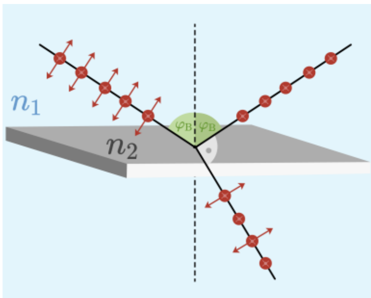
6.2 Polarizing filter

$$I = I_0 \cos^2(\varphi)$$



Unpolarized light passes through a polarizing filter and becomes linearly polarized. The linearly polarized light has the **initial intensity** I_0 [W/m^2]. Then it passes through a second linear polarization filter which is rotated by the **angle** φ [rad] with respect to the first one. And I [W/m^2] is the **intensity** after passing the second polarizing filter (*Malus' law*).

$$\varphi_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$



If *unpolarized* light falls at the **Brewster angle** (**polarization angle**) φ_B [rad] on the interface of two dielectric media (for example, air and glass) with **refractive indices** n_1 [−] and n_2 [−], then the reflected light is completely *linearly* polarized *perpendicular* to the plane of incidence (s-polarization). The transmitted light, on the other hand, is only partially linearly polarized.

6.3 Light passes through a single slit

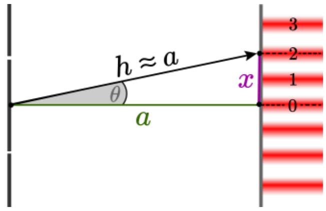
$$\sin(\varphi) = \frac{m\lambda}{d}$$

The light of **wavelength** λ [m] passes through the single slit of **width** d [m] and generates an interference pattern on a detector screen. Here, φ [rad] is the **angle** between the *optical axis* (which is perpendicular to the slit) and the the *direction of observation* (hypotenuse).

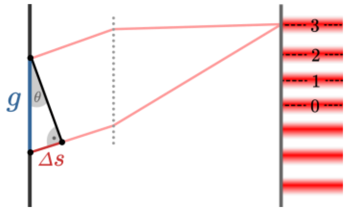
- For the values $m = 1, 2, 3, \ldots$ [−] the angle φ indicates the position of different *minima* (dark fringes).
- For the values $m = 0, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$ [−] the angle indicates the position of different *maxima* (bright fringes).

6.4 Light passes through a double-slit

$$\frac{m\lambda}{g} \approx \frac{x}{a}$$



The light of **wavelength** λ [m] passes through the double slit with **slit distance** g [m] and creates an interference pattern on a detector screen, which is at **distance** a [m] from the double slit. A bright mth fringe is located at **distance** x [m] from the main maximum (0th fringe). Here, the **path difference** $\Delta s = m\lambda$ [m] is a multiple of the wavelength when a *bright* fringe is considered. With $m = 0, 1, 2, 3 \ldots$ [−].



$$\frac{(m - 1/2)\lambda}{g} \approx \frac{x}{a}$$

The formula with the **path difference** $\Delta s = (m - 1/2)\lambda$ is used when the **distance** x [m] from the main maximum to a *dark* fringe is considered. Here $m = 1, 2, 3 \ldots$ [−].

6.5 Optical (diffraction) grating

$$\sin(\varphi) = \frac{m\lambda}{g}$$

The light of **wavelength** λ [m] passes through an *diffraction grating* with **grating constant** g [m] and produces an interference pattern on a detector screen. Here, φ [rad] is the **angle** between the *optical axis* (perpendicular to the grating) and the *direction of observation* (hypotenuse). For **numbers** $m = 0, 1, 2, 3, \ldots$ [−], the angle φ indicates the position of various *maxima* (distinct bright fringes).

$$\frac{\lambda}{\Delta\lambda} = mN$$

If a *non-monochromatic* light is used (it contains several wavelengths), then an interference pattern occurs separately for each **wavelength** λ [m]. Here, $\Delta\lambda = \lambda_1 - \lambda$ [m] is the **wavelength difference** of two *adjacent* wavelengths whose spectral lines can just be perceived as separate. The ratio $\lambda/\Delta\lambda$ [−] defines the **resolution** of an optical grating. And N [−] is the **number of grating slits that are illuminated**. Here, $m = 0,1,2, \ldots$ [−] indicates the **order of a maximum**.

6.6 Thin-film interference

$$\Delta s = 2d\sqrt{n^2 - \sin(\varphi)^2} \qquad \text{with } n' > n$$

$$\Delta s = 2d\sqrt{n^2 - \sin(\varphi)^2} - \frac{\lambda}{2} \qquad \text{with } n' < n$$

The light of **wavelength** λ [m] falls at **angle** φ [rad] (angle between the light beam and perpendicular) on a thin first layer of thickness d [m]. This thin layer has **refractive index** n [−] and is placed on a second layer which has **refractive index** n' [−]. The light beam reflected at the first layer interferes with the light beam reflected at the second layer.

- The **path difference** $\Delta s = m\lambda$ [m] with order number $m = 0, 1, 2, 3 \ldots$ occurs when observing a *bright* interference fringe.
- The **path difference** $\Delta s = (m - 1/2)\lambda$ [m] with order number $m = 1, 2, 3 \ldots$ occurs when observing a *dark* interference fringe.

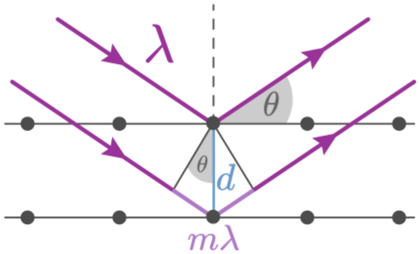
6.7 Newton rings

$$r_m = \sqrt{\frac{m\lambda r}{n}}, \qquad m = 1, 2, 3, \ldots$$

A *plano-convex lens* with the **radius of curvature** r [m] lies with the curved side on a flat glass plate. The two are in a medium with the **refractive index** n [−] (for example for air $n = 1$). The lens is irradiated vertically with a light of **wavelength** λ [m]. Light and dark interference rings are formed around the point of contact between the lens and the glass plate. Here r_m [m] is the **radius** of the m-th dark interference ring. For example, r_1 is the radius of the first interference ring.

6.8 Reflection at crystals (Bragg's law)

$$m\lambda = 2d\sin(\theta)$$

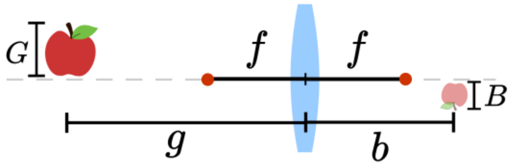


Bragg's law describes the reflection of light of **wavelength** λ [m] at two crystal planes (lattice planes) located at a **distance** (**lattice constant**) d [m] from each other. The **glancing angle** θ [rad] is the angle at which an interference *maximum* occurs. The **diffraction order** m [−] is a natural number and specifies the m-th maximum.

6.9 Lenses

$$f = \frac{bg}{g + b} \qquad D = \frac{1}{f}$$

$$M = \frac{B}{G} = \frac{b}{g}$$



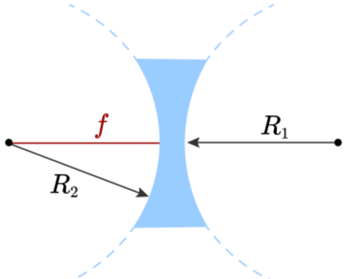
Thin lens equation indicates the relationship between the **focal length** f [m] of a thin lens, the **object width** g [m] and the **image width** b [m]. The **optical power** D [dpt = 1/m] of a lens is the reciprocal of the focal length f and is usually given in *diopter unit* (dpt).

The **magnification** M [−] is the ratio of the **image size** B [m] to the **object size** G [m]. The magnification can also be calculated as the ratio of the image width b to the object width g.

$$f = \frac{n_o}{n_i - n_o}\left(\frac{1}{R_1} - \frac{1}{R_2} - \frac{(n_i - n_o)d}{n_i R_1 R_2}\right)^{-1}$$

The **focal length** f [m] of a thick lens depends on the **refractive index** n_o [−] **of the medium outside** the lens and the **refractive index** n_i [−] **of the lens material**. The focal length also depends on the **curvature radius** R_1 [m] **of the right side** of the lens and the **curvature radius** R_2 [m] **of the left side** of the lens, as well as the **thickness** d [m] of the lens (measured along the optical axis).

$$f = \frac{n_o}{n_i - n_o}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}$$



The **focal length** f [m] of a *biconcave lens* depends on the **refractive index** n_o [−] of the medium *outside* the lens and on the **refractive index** n_i [−] *inside* the lens. The focal length also depends on the **radius of curvature** R_1 [m] at the right side of the lens and on the **radius of curvature** R_2 [m] at the left side of the lens.

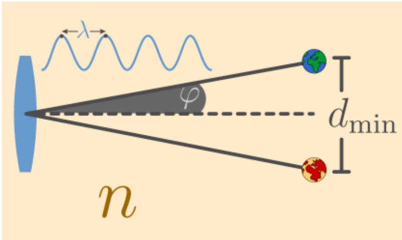
6.10 Telescopes and microscopes

$$A_N = n \sin(\varphi)$$

Numerical aperture A_{N} [−] describes the medium between the *enlarger* (telescope, microscope) and the *object* to be observed through the medium with **refractive index** n [−] and describes **half the aperture angle** φ [rad] of the objective of the enlarger.

$$d_{\min} = 0.61 \cdot \frac{\lambda}{A_{\mathrm{N}}}$$

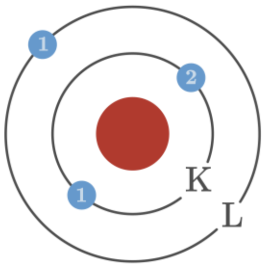
$$d_{\min} = 0.61 \cdot \frac{\lambda}{n \sin(\varphi)}$$



Rayleigh criterion describes the **minimum distance** $d_{\min}$ [m] of two objects which would still be distinguishable if the light **wavelength** λ [m] is used for their observation and a medium with the **refractive index** n [−] is present between the object and the enlarger. Here, φ [rad] is **half the aperture angle**.

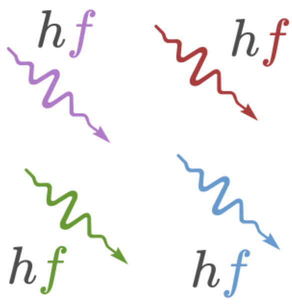
7. QUANTUM PHYSICS

Uncertainty principle, superposition and entanglement.



7.1 Photon

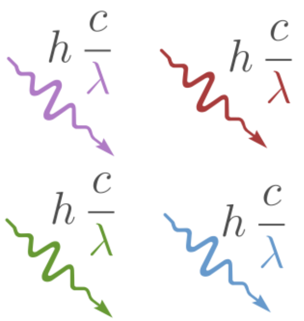
$$W_\mathrm{p} = hf \quad W_n = nhf \quad W_\mathrm{m} = N_\mathrm{A}hf$$



The **photon energy** W_p [J] of a *single* photon (light particle), depends on the light **frequency** f [Hz].

- W_n [J] is the **energy of** n [−] **photons**.
- W_m [J/mol] is the **energy of one mole of photons**. Here N_A [1/mol] is the **Avogadro constant**.

$$W_\mathrm{p} = h\frac{c}{\lambda} \quad W_n = nh\frac{c}{\lambda} \quad W_\mathrm{m} = N_\mathrm{A}h\frac{c}{\lambda}$$



The photon energy W_p can also be expressed with the **speed of light** c [m/s] and the light **wavelength** λ [m].

$$p = \frac{h}{\lambda} \quad p = \frac{hf}{c}$$

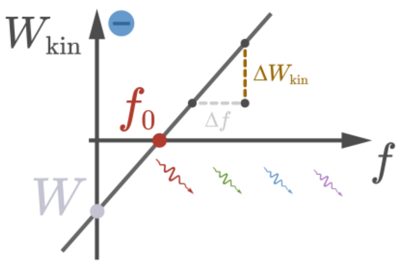
The **momentum** p [kg · m/s] of *one* photon as a function of the light **wavelength** λ [m] or as a function of the light **frequency** f [Hz].

7.2 Photoelectric effect

$$W_\mathrm{p} = W_\mathrm{kin} + W$$

$$hf = \frac{1}{2}m_\mathrm{e}v^2 + hf_0$$

$$hf = eU_\mathrm{G} + hf_0$$



The *Einstein equation* describes the energy conservation in the *photoelectric effect*. The **photon energy** W_p [J] of a photon that falls onto a metal plate with the **work function** W [J] can eject an electron with the **kinetic energy** W_kin [J]. Here, the photon has the **frequency** $f = c/\lambda$ [Hz], the metal plate has the **threshold frequency** f_0 [Hz], and the ejected electron with **mass** m_e [kg] has the (**maximum**) **velocity** v [m/s]. The kinetic energy can also be determined using the **stopping voltage** U_G [V] and the **elementary charge** e [C], when the electron is ejected from a metal electrode of a plate capacitor and reaches the opposite electrode. The Einstein equation can be interpreted as a linear function $W_\mathrm{kin}(f)$ [J] in an *energy-frequency graph*, from whose slope the **Planck constant** h [Js] can be determined.

$$W = hf_0 \quad W = h\frac{c}{\lambda_0}$$

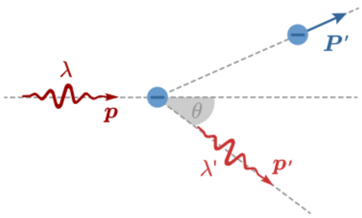
The **work function** W [J] of a material can be calculated either using the **threshold frequency** f_0 [Hz] or the **threshold wavelength** λ_0 [m].

Light color	Frequency f in THz	Wavelength λ in nm	Energy W_p in eV	Energy W_p in 10^{-19} J
Red	400 to 462	650 to 750	1.65 to 1.91	2.6 to 3.1
Yellow	513 to 522	575 to 585	2.12 to 2.16	3.4 to 3.5
Green	522 to 612	490 to 575	2.16 to 2.53	3.5 to 4.1
Blue	612 to 714	420 to 490	2.53 to 2.95	4.1 to 4.7
Violet	714 to 789	380 to 420	2.95 to 3.26	4.7 to 5.2

7.3 Compton effect

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos(\theta))$$

$$\Delta\lambda = \lambda_\text{C}(1 - \cos(\theta))$$



The photon **wavelength** λ [m] **before** the collision with a stationary particle of **mass** m [kg] changes to the **wavelength** λ' [m] **after** the collision. Here, θ [rad] is the scattering angle after the collision, $\lambda_\text{C} = \frac{h}{mc}$ [m] is the **Compton wavelength**, and $\Delta\lambda = \lambda' - \lambda$ [m] is the **difference in wavelengths**.

7.4 Casimir effect

$$F = \frac{\pi^2 \hbar c}{240} \frac{A}{d^4} \quad \Pi = \frac{\pi^2 \hbar c}{240} \frac{1}{d^4}$$

The **Casimir force** F [N] is the attractive force between two uncharged metal plates of inner surface area A [m^2] that are in close proximity at a **distance** d [m] from each other *in a vacuum*. This attraction between the plates is called the *Casimir effect* and can be interpreted as a result of vacuum fluctuations. The vacuum exerts a **pressure** Π [Pa] on the plates from the outside. Here, $\hbar$ [Js] is the **reduced Planck's constant** and c [m/s] is the **speed of light**.

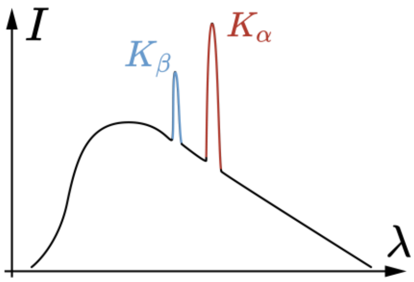
7.5 Bremsstrahlung (deceleration radiation)

$$f_g = \frac{eU_b}{h} \quad \lambda_g = \frac{hc}{eU_b} \quad W_g = eU_b$$

A decelerated electron generates *bremsstrahlung* (radiation) of various frequencies. The electron passes through the **acceleration voltage** U_b [V], and thus gain a certain amount of **kinetic energy**. If all of the kinetic energy is converted into a photon (bremsstrahlung), then the photon has the **maximum possible energy** (**threshold energy**) W_g [J], **minimum threshold wavelength** λ_g [m], and **maximum threshold frequency** f_g [Hz].

7.6 Characteristic lines in the X-ray spectrum (Moseley's law)

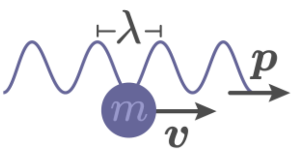
$$\lambda_{K_\alpha} = \frac{4}{3R_y(Z - 1)^2}$$



An anode is bombarded with fast electrons. The electrons are decelerated and generate X-rays with a *continuous spectrum* in an intensity-wavelength graph, as well as *characteristic lines* (peaks in the spectrum). The **wavelength** λ_{K_α} [m] belongs to the K_α line in the X-ray spectrum. Here, Z [−] is the **number of protons** (**atomic number**) of the atom type from which the anode material consists, and $R_y = 1.097 \cdot 10^7/\text{m}$ is the **Rydberg constant**.

7.7 De-Broglie wavelength

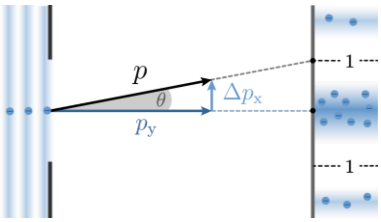
$$\lambda = \frac{h}{mv} \quad \lambda = \frac{h}{p}$$



De-Broglie wavelength (**matter wavelength**) λ [m] of a (quantum) particle of **mass** m [kg] moving with **velocity** v [m/s]. Here $p = mv$ [kg · m/s] is the **momentum** of the particle.

7.8 Heisenberg uncertainty principle

$$\Delta p \geq \frac{h}{4\pi\Delta x}$$



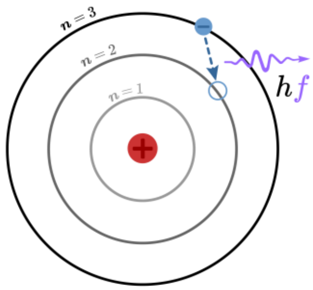
The **position** x [m] of a particle is uncertain and lies within the range of $x - \Delta x$ to $x + \Delta x$. According to the *Heisenberg's* *uncertainty principle*, the **momentum** p [$\mathrm{kg \cdot m/s}$] of the particle must have at least the **momentum uncertainty** Δp [$\mathrm{kg \cdot m/s}$], which is fundamentally determined by the **position uncertainty** Δx [m] and the **Planck's constant** h [Js]. The momentum of the particle lies in the interval between $p - \Delta p$ and $p + \Delta p$, and this interval cannot be reduced.

$$\Delta W \Delta t \geq \frac{\hbar}{2}$$

Besides the position-momentum uncertainty, there is also an *energy-time uncertainty*. Here ΔW [J] is the **energy deviation** of the energy W of the particle and Δt [s] the **time deviation** of the time t at which the energy is measured.

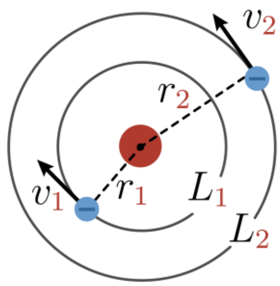
7.9 Bohr model of an atom

$$hf = W_m - W_n$$



An electron in an atom can only assume discrete **energy values** W_m [J] and W_n [J]. Here the **principal quantum numbers** m [−] and $n = 1, 2, 3, \ldots$ [−] define two different energies of the electron, with $m > n$. When an electron moves from the m-th energy state to the n-th energy state, the atom emits a **photon of energy** hf [J] and **light frequency** f [Hz].

$$m_e r_n v_n = n\hbar \quad L_n = n\hbar \quad U_n = n\lambda_{\mathrm{dB}}$$



The electrons can orbit the nucleus at the **distance** r_n [m] with the **velocity** v_n [m/s] only on certain orbits without loss of energy. Here $\hbar = h/2\pi$ [Js] is the **reduced Planck's constant** and m_e [kg] is the **rest mass of an electron**. The **angular momentum** L_n [$\mathrm{Nm \cdot s = Js}$] of an electron in the nth state is a multiple of Planck's constant $\hbar$. The **circumference** U_n [m] of the electron orbit around the nucleus is a multiple of its **De Broglie wavelength** λ_{dB} [m].

$$r_B = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \approx 0.529 \cdot 10^{-10}\ \mathrm{m} \qquad \mu_B = \frac{e\hbar}{2m_e} \approx 9.274 \cdot 10^{-24}\ \frac{\mathrm{J}}{\mathrm{T}}$$

Bohr radius r_B [m] is composed only of physical constants and specifies the **radius of the electron orbit** in the ground state of the H atom. **Bohr magneton** μ_B [J/T] is also composed only of physical constants and specifies the **magnetic dipole moment** of an electron with the **azimuthal quantum number** $l = 1$.

$$N = 2 \cdot (1 + 2^2 + 3^2 + \cdots + n^2) = \frac{n(n+1)(2n+1)}{3}$$

The **number** N $[-]$ **of orbitals** to the n-th **principal quantum number**. For example, the electrons of a hydrogen atom in its $n = 3$ lowest shells (K, L und M shells), can occupy $N = 28$ states. The number of electrons in **only one** shell, on the other hand, is $N_n = 2n^2$. The factor 2 takes into account the spin of the electron.

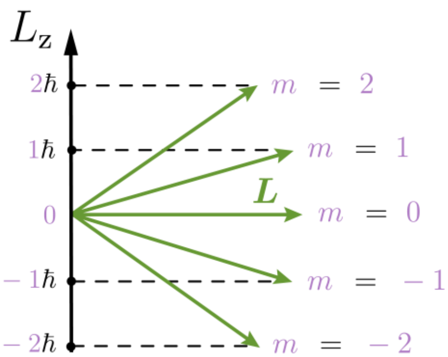
7.10 Angular momentum in quantum mechanics

$$L = \sqrt{l(l+1)\hbar^2}$$

Magnitude of the angular momentum L $[\mathrm{Js}]$ of a quantum mechanical particle (for example an electron). Here, the **angular momentum quantum number** l $[-]$ takes on non-negative values: $l = 0,\ 1/2,\ 1,\ 3/2,\ 2, \ldots$

- **Orbital angular momentum quantum numbers**
  $l = 0, 1,\ 2, \ldots$
- **Spin quantum numbers**
  $l = 1/2,\ 3/2,\ 5/2, \ldots$

$$L_z = m\hbar$$

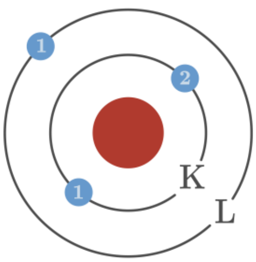


Magnitude L_z $[\mathrm{Js}]$ **of the angular momentum in z-space direction** is a multiple of the Planck's constant $\hbar$. Here, m $[-]$ is the **magnetic quantum number** of the angular momentum and can only take integer values between -l and l and in +1 steps. For example for $l = 2$: $m = -2, -1, 0, 1, 2$.

7.11 Quantum numbers and electron configurations

Principal quantum number n $[-]$

- $n = 1$ is the **K** shell.
- $n = 2$ is the **L** shell.
- $n = 3$ is the **M** shell.
- $n = 4$ is the **N** shell.
- …



The n-th shell can be occupied by a *maximum* of $2n^2$ electrons.

Azimuthal quantum number l $[-]$ takes non-negative integers from 0 to $n-1$.

- $l = 0$ is the **s** orbital.
- $l = 1$ is the **p** orbital.
- $l = 2$ is the **d** orbital.
- $l = 3$ is the **f** orbital.
- …
- $l = n - 1$

Magnetic quantum number m $[-]$ takes integers from $-l$ to $+l$.

- $m = -l$
- ...
- $m = l$

For example, for $l = 2$, m can take the values -2, 1, 0, 1, 2.

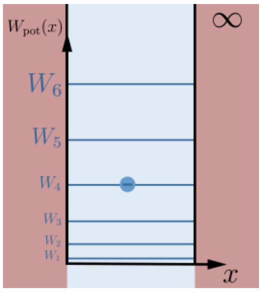
Spin quantum number s $[-]$ takes the values $s = -\frac{1}{2}$ and $s = \frac{1}{2}$.

Label	Principal quantum number n	Azimuthal quantum number l	Magnetic quantum number m	Spin quantum number s
1s	1 (K)	0 (s)	0	-1/2, 1/2
2s 2p	2 (L)	0 (s) 1 (p)	0 -1, 0, 1	-1/2, 1/2 -1/2, 1/2
3s 3p 3d	3 (M)	0 (s) 1 (p) 2 (d)	0 -1, 0, 1 -2, -1, 0, 1, 2	-1/2, 1/2 -1/2, 1/2 -1/2, 1/2
4s 4p 4d 4f	4 (N)	0 (s) 1 (p) 2 (d) 3 (f)	0 -1, 0, 1 -2, -1, 0, 1, 2 -3, -2, -1, 0, 1, 2, 3	-1/2, 1/2 -1/2, 1/2 -1/2, 1/2 -1/2, 1/2

Atom	Electron configuration	Atom	Electron configuration
Hydrogen (H)	$1s^1$	**F**luorine (F)	$[He]2s^2 2p^5$
Helium (He)	$1s^2$	**Ne**on (Ne)	$[He]2s^2 2p^6$
Lithium (Li)	$[He]2s^1$	**Na**trium (Na)	$[Ne]3s^1$
Beryllium (Be)	$[He]2s^2$	**M**a**g**nesium (Mg)	$[Ne]3s^2$
Boron (B)	$[He]2s^2 2p^1$	**Al**uminium (Al)	$[Ne]3s^2 2p^1$
Carbon (C)	$[He]2s^2 2p^2$	**Si**licium (Si)	$[Ne]3s^2 2p^2$
Nitrogen (N)	$[He]2s^2 2p^3$	**P**hosphor (P)	$[Ne]3s^2 2p^3$
Oxygen (O)	$[He]2s^2 2p^4$	**Ar**gon (Ar)	$[Ne]3s^2 3p^6$

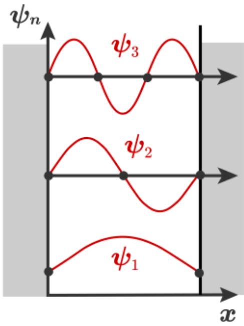
The *superscript* in the electron configuration indicates the *number of electrons* in the corresponding orbital.

7.12 Quantum particle in a box



$$W_n = \frac{h^2}{8mL^2}n^2 \quad W_1 = \frac{h^2}{8mL^2}$$

Energy W_n [J] of a quantum mechanical particle of **mass** m [kg] confined in an infinite potential box of **length** L [m]. Here, n [−] is a **quantum number** that numbers the allowed energies that the particle can occupy. W_1 is the **ground state energy** with $n = 1$.

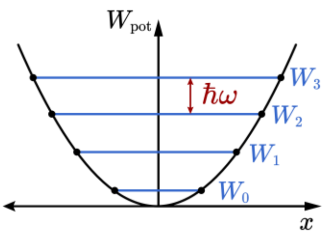


$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$$

W**ave function** $\psi_n(x)$ $\left[1/\sqrt{\text{m}}\right]$ of the particle inside the potential box in the n-th state. Here $\psi_0(x)$ is the **ground state wave function**.

7.13 Quantum mechanical harmonic oscillator

$$W_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad W_0 = \frac{1}{2}\hbar\omega$$



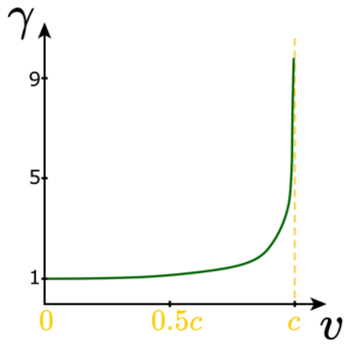
Energy W_n [J] of a quantum mechanical particle (for example an electron) in the n-th state in the **parabolic potential** $W_{\text{pot}}(x)$ [J]. Here W_0 [J] is the **ground state energy**. ω [rad/s] is the characteristic **angular frequency** of the harmonic oscillator and indicates how fast the particle oscillates. And $\hbar$ [Js] is the **reduced Planck constant**.

8. RELATIVISTIC MECHANICS

When classical mechanics reaches its limits...

8.1 Lorentz (gamma) factor

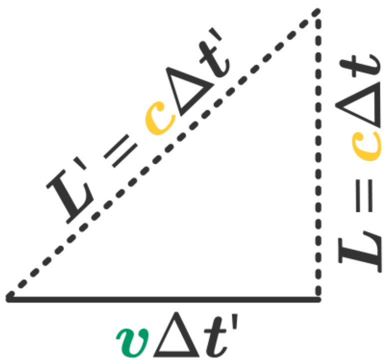
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Lorentz factor (**Gamma factor**) γ [–] is used in relativistic equations and gives the factor by which, for example, time t' [s] in a moving reference frame A differs from time t [s] in a resting reference frame B: $t' = \gamma t$. The Lorentz factor depends on the **velocity** v [m/s] of the reference frame moving relative to a fixed system at rest. The Lorentz factor is always *greater than* 1. Here c [m/s] is the **speed of light**.

8.2 Time dilation

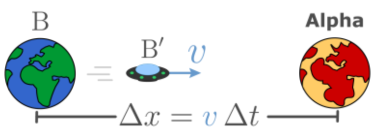
$$\Delta t' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta t \quad \Delta t' = \gamma \Delta t$$



Time $\Delta t'$ [s] between two events, which passes on the moving clock from the point of view of an observer at rest. For the observer at rest, on the other hand, the **time** Δt [s] has passed. The rest observer sees the moving clock passing by with the **velocity** v [m/s]. The **Lorentz factor** γ [–] is always greater than 1.

8.3 Length contraction

$$\Delta x' = \sqrt{1 - \frac{v^2}{c^2}}\Delta x \quad \Delta x' = \frac{1}{\gamma}\Delta x$$

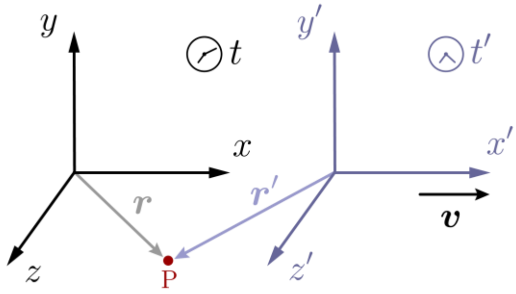


Length $\Delta x'$ [m] of a body (for example a rod) measured by an observer moving relative to this body at the **speed** v [m/s]. Here, Δx [m] is the **rest length** of the body. The **Lorentz factor** γ [–] is always greater than 1.

8.4 Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$y = y' \qquad z = z'$$

Spatial coordinates (**position**) x, y, z [m] of an inertial system A transform into the **spatial coordinates** x', y', z' [m] of another inertial system B by the above equations. Here t [s] is the **time coordinate in the system A** and t' [s] is the **time coordinate in system B**. System B moves relative to system A with the **velocity** v [m/s] along the x-direction.

8.5 Relativistic addition of velocities

$$u = \frac{u' + v}{1 + \frac{v}{c^2}u'}$$

Consider two inertial systems S and S′.

- **Velocity** u [m/s] of the system S.
- **Velocity** u' [m/s] of the system S′.
- System S moves with **velocity** v [m/s] relative to system S′.

8.6 Relativistic mass

$$m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic mass $m(v)$ [kg] of a body moving with **velocity** v [m/s] relative to the observer at rest. Here m_0 [kg] is the **rest mass** of this body.

8.7 Equivalence of mass and energy

$$W = mc^2 \quad W_0 = m_0c^2 \quad W_{\text{kin}} = (m - m_0)c^2$$

Total energy W [J] of a system A moving relative to the system at rest with **relative velocity** v [m/s]. Here m [kg] is the **relativistic mass** of the system A, which depends on the relative velocity v. And m_0 [kg] is the **rest mass** of system A - that is, its mass when system A is not moving relative to the rest system. The system A has the **rest energy** W_0 [J]. Its **relativistic kinetic energy** W_{kin} [J] is the total energy W minus the rest energy W_0.

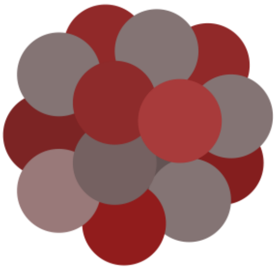
8.8 Relativistic energy-momentum relation

$$W = \sqrt{W_0^{\,2} + p^2c^2}$$

Relativistic total energy W [J] of a system, which is valid also at large velocities. Here W_0 [J] is its **rest energy** and p [kg · m /s] its **relativistic momentum**.

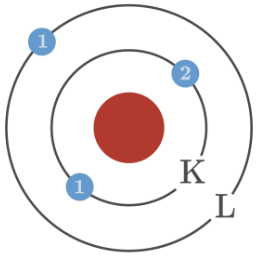
9. ATOM AND NUCLEAR PHYSICS

A look into the heart of the world.



9.1 Atomic mass

$$m = A_\mathrm{r}\mathrm{u} \quad m = \frac{A_\mathrm{r}}{N_\mathrm{A}} \cdot 1\,\frac{\mathrm{g}}{\mathrm{mol}}$$

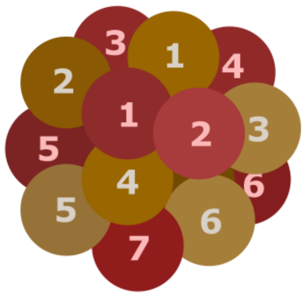


Absolute mass m [kg] of an atom in **unified atomic mass unit** (**Dalton**) [u]. To convert the atomic mass in atomic mass unit into kg, the **relative mass** A_r [−] must be multiplied by $1.660 \cdot 10^{-27}$ kg. Mostly only the *nucleon number* (number of protons and neutrons) is taken for the relative mass A_r, because the electrons are much lighter and thus in most cases are not considered. You can find the relative mass of different atoms in the periodic table of the elements. Here N_A [1/mol] is the **Avogadro constant**.

Atom	Absolute mass m	Relative mass A_r
Helium (He)	$33.2 \cdot 10^{-28}$ kg	2
Aluminium (Al)	$215.8 \cdot 10^{-28}$ kg	13
Xenonium (Xe)	$896.4 \cdot 10^{-28}$ kg	54

9.2 The Nucleus

$$A = Z + N$$

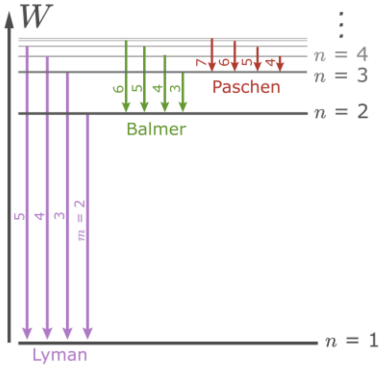


Nucleon number (**mass number**) A [−] is the **number of protons** (**Atomic number**) Z [−] plus the **number of neutrons** N [−] in a nucleus. The nucleon number is different for different elements of the periodic table.

$$r \approx r_\mathrm{p}\sqrt[3]{A}$$

Formula to estimate the **radius** r [m] **of a nucleus**. The nucleus radius depends on the **radius of the proton** $r_\mathrm{p} = 1.4 \cdot 10^{-15}$ m and the **nucleon number** A [−].

9.3 The hydrogen (H) atom



$$W = \frac{13.6\ \mathrm{eV}}{n^2}$$

Binding energy W $[-]$ is the energy necessary to knock the electron with **principal quantum number** n $[-]$ out of the H atom.

$$\lambda = \frac{1}{R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)}$$

Light wavelength λ $[\mathrm{m}]$ used to transport the electron in the H atom from the **n-th energy** level to the higher **m-th energy** level. Here, R $[1/\mathrm{m}]$ is the **Rydberg constant**. The **energy** W_p $[\mathrm{J}]$ **of a photon** of wavelength λ can be calculated as follows:

$$W_\mathrm{p} = h\frac{c}{\lambda}$$

9.4 Mass of an atom and of a nucleus

$$m_\mathrm{A} = m_\mathrm{K} + Zm_\mathrm{e} - \frac{W_\mathrm{b}}{c^2} \qquad m_\mathrm{A} \approx m_\mathrm{K} + Zm_\mathrm{e}$$

The **mass** m_A $[\mathrm{kg}]$ **of an atom** is composed of the **mass** m_K $[\mathrm{kg}]$ **of the nucleus** and the mass m_e $[\mathrm{kg}]$ of Z $[-]$ **electrons** minus the mass W_b/c^2 $[\mathrm{kg}]$. The subtracted mass results from the **binding energy** W_b $[\mathrm{J}]$ of all Z electrons. The contribution by the electron binding energy is very small and can be neglected in most cases.

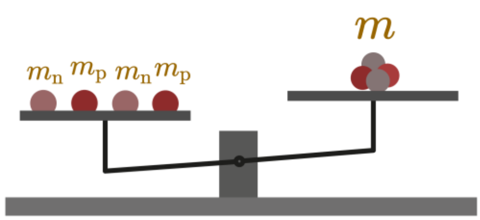
$$W_\mathrm{b} \approx 15.73\ \mathrm{eV} \cdot Z^{\frac{7}{3}}$$

With the *Thomas-Fermi equation* the **electron binding energy** W_b $[-]$ of Z electrons can be estimated. For the H-atom: $W_\mathrm{b} = 13.6\ \mathrm{eV}$.

9.5 Mass defect and binding energy

$$W = \Delta m c^2$$

$$\Delta m = Z m_\mathrm{p} + N m_\mathrm{n} - m$$



Z [−] **protons** with **proton mass** m_p [kg] and N [−] **neutrons** with **neutron mass** m_n [kg] are combined to a nucleus. The *mass defect* states that the **mass difference** Δm [kg] of the individual nucleons and the **mass** m [kg] **of the nucleus** is not zero. This mass difference is in the **binding energy** W [J] of the nucleus.

$$W = W_1 A - W_2 \sqrt[3]{A^2} - W_3 \frac{Z^2}{\sqrt[3]{A}} - W_4 \frac{(A - 2Z)^2}{A} + W_5 \frac{\delta}{\sqrt[4]{A^3}}$$

The **binding energy** W [J] of a nucleus can be calculated with the *semi-empirical mass formula* if the **nucleon number** A [−] (sum of neutrons and protons) is above 30 (deviation less than 1%). Here Z [−] is the **number of protons** (**atomic number**) and the **factor** δ [−] can take three different values:

- $\delta = 1$ if the proton number and neutron number are *both even.*
- $\delta = -1$ if the proton number and neutron number are *both odd.*
- $\delta = 0$ if *proton number is odd* and *neutron number is even* or vice versa.

The **energy constants** have the following values: $W_1 = 15.75$ MeV, $W_2 = 17.8$ MeV, $W_3 = 0.71$ MeV, $W_4 = 23.7$ MeV and $W_5 = 34$ MeV.

9.6 Reaction energy (Q value)

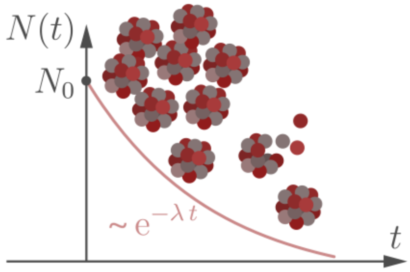
$$Q = (m_{A0} + m_{B0})c^2 - (m'_{A0} + m'_{B0})c^2$$

Two nuclei A and B collide. The **reaction energy** Q [J] is the difference of the rest energies before the reaction and after the reaction.

- m_{A0} [kg] the **rest mass** of the nucleus A *before* the collision.
- m'_{A0} [kg] the **rest mass** of the nucleus A *after* the collision.
- m_{B0} [kg] is the **rest mass** of the nucleus B *before* the collision.
- m'_{B0} [kg] the **rest mass** of the nucleus B *after* the collision.

9.7 Radioactive decay

$$N(t) = N_0 \mathrm{e}^{-\lambda t}$$



Number $N(t)$ $[-]$ **of not yet decayed nuclei** at **time** t $[\mathrm{s}]$ of a radioactive sample. **Before the decay**, at time $t = 0$, there were N_0 $[-]$ **not decayed nuclei**. The **decay constant** λ $[1/\mathrm{s}]$ indicates the decay probability per unit time. To put it graphically, the decay constant determines how fast a nuclide (radioactive material) decays. Different nuclides decay at different rates.

$$t_{\mathrm{h}} = \ln(2)\lambda$$

Half-life t_{h} $[\mathrm{s}]$ is the time after which the initial inventory N_0 of atomic nuclei has decayed to half. The half-life depends on the **decay constant** λ $[1/\mathrm{s}]$. The reciprocal of the decay constant is the **average lifetime** $\tau = 1/\lambda$ $[\mathrm{s}]$ of a nuclide.

$$A = \lambda N(t) \quad A = -\frac{\Delta N}{\Delta t}$$

Activity A $[\mathrm{Bq} = 1/\mathrm{s}]$ of a radioactive sample indicates how fast the atoms of the radioactive sample decay. The activity depends on the **decay constant** λ $[1/\mathrm{s}]$ and on the **number** $N(t)$ $[-]$ **of not yet decayed nuclei**. Here $\Delta N = N_2 - N_1 < 0$ $[-]$ is the number of nuclei decayed within the **time** Δt $[\mathrm{s}]$.

Isotop		Half-life t_{h}	Decay constant λ
Beryllium (Be)	$^{13}_{4}\mathrm{Be}$	$2.7 \cdot 10^{-21}$ s	$3.9 \cdot 10^{-21}$ 1/s
Radonium (Ra)	$^{220}_{86}\mathrm{Rn}$	56 s	80.8 1/s
Natrium (Na)	$^{22}_{11}\mathrm{Na}$	$8.2 \cdot 10^{8}$ s (2.6 years)	$11.8 \cdot 10^{8}$ 1/s
Cobaltium (Co)	$^{60}_{27}\mathrm{Co}$	$1.7 \cdot 10^{9}$ s (5.3 years)	$2.4 \cdot 10^{9}$ 1/s
Cesium (Cs)	$^{137}_{55}\mathrm{Cs}$	$9.5 \cdot 10^{9}$ s (30.1 years)	$13.7 \cdot 10^{9}$ 1/s
Radium (Ra)	$^{226}_{88}\mathrm{Ra}$	$5 \cdot 10^{10}$ s (1600 years)	$7.2 \cdot 10^{10}$ 1/s
Uranium (U)	$^{235}_{92}\mathrm{U}$	$2.2 \cdot 10^{16}$ s (700 million years)	$3.2 \cdot 10^{16}$ 1/s
Thorium (Th)	$^{232}_{90}\mathrm{Th}$	$4.4 \cdot 10^{17}$ s (14 billion years)	$6.3 \cdot 10^{17}$ 1/s

9.8 Ion dose, absorbed and equivalent dose

$$J = \frac{\Delta Q}{\Delta m} \quad D = \frac{\Delta W}{\Delta m} \quad H = D\omega_R$$

A substance is irradiated with radioactive radiation. The **ion dose** J $[\mathrm{C/kg}]$ is the amount of **charge** ΔQ $[\mathrm{C}]$ per **mass** Δm $[\mathrm{kg}]$ of the irradiated substance produced by ionization. **Absorbed dose** D $[\mathrm{Gy} = \mathrm{J/kg}]$ is the **energy** ΔW $[\mathrm{J}]$ **absorbed** by the irradiated substance per mass. Since the type of decay (alpha, beta, gamma) differs in its biological effect, the absorbed dose is multiplied by the **radiation weighting factor** ω_R $[-]$ to obtain the **equivalent dose** H $[\mathrm{Sv} = \mathrm{J/kg}]$. This quantity is more suitable for making a statement about the effect of radioactive radiation on organisms.

- $\omega_R = 1$ is for *gamma* and *beta* decay.
- $\omega_R = 20$ is for *alpha* decay.

9.9 Alpha, beta and gamma decay

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{X} + {}^{4}_{2}\mathrm{He}$$

Alpha decay occurs when the radioactive nucleus of **chemical species** X with **nucleon number** A $[-]$ (number of neutrons and protons) and **proton number** Z $[-]$ is too heavy. Thereby the atomic nucleus emits a **helium atom** ${}^{4}_{2}\mathrm{He}$ with 4 nucleons and 2 protons. The decayed atomic nucleus has now 2 neutrons and 2 protons less.

$${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z+1}^{A}\mathrm{X} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

Beta-minus decay occurs when the ratio of the number of neutrons and protons is unfavorable. In this case, a neutron n decays into a **proton** p, an **electron** e^- and an **electron antineutrino** $\bar{\nu}_\mathrm{e}$. The **chemical species** X emits the electron and an electron antineutrino as radioactive radiation and has now one proton more.

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-1}_{Z-1}\mathrm{X} + \mathrm{e}^+ + \nu_\mathrm{e}$$

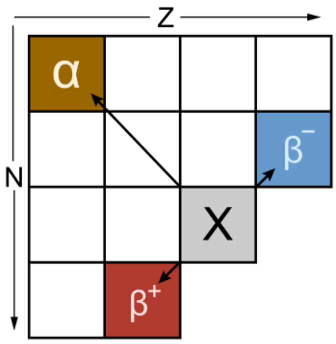
$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$$

Beta-plus decay also occurs when the ratio of the number of neutrons and protons is unfavorable. Here, a **proton** p decays into a **neutron** n, a **positron** e^+ and an **electron neutrino** ν_e. The **chemical species** X emits the positron and an electron neutrino as radioactive radiation and now has one proton less.

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X \rightarrow \gamma$$

Gamma decay occurs when the atomic nucleus is excited. The **excited nucleus** $^{A}_{Z}X^{*}$ of **chemical species** X emits a gamma quantum γ (high frequency electromagnetic radiation) and falls into an energetically lower state.

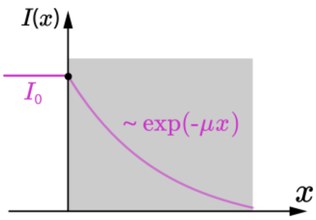
Using a **table of nuclides** with the horizontal axis (proton number) and vertical axis (neutron number), you can determine the daughter nuclide (nuclide X after decay).



- After the **alpha decay**, you get to the daughter nuclide, starting from the mother nuclide X, if you go two boxes up and two boxes to the left in the table of nuclides.
- After the **beta-minus decay**, go one box up and one box to the right.
- After the **beta-plus decay**, go one box to the left and one box down.

9.10 Absorption law

$$I(x) = I_0 \mathrm{e}^{-\mu x}$$



A material of **thickness** x [m] is irradiated with radioactive radiation from one side. On the other side there is a *Geiger-Müller counter* which measures the **count rate** $I(x)$ [1/s]. If the counter is used *without* the material, then the **count rate** is I_0 [1/s]. Here μ [1/m] is the absorption coefficient.

10. ASTROPHYSICS

Formulas describing the macrocosm.

10.1 Earth parameters

Average radius:	6370 km
Mass:	$5.97 \cdot 10^{24}$ kg
Density:	$5500 \frac{kg}{m^3}$
Gravitational acceleration:	$9.8 \frac{m}{s^2}$
Rotation around its own axis:	1 rotation in 24 hours
Rotation around the center of gravity of the Earth-Moon system:	1 revolution in 27 days
Distance to the sun:	on average $150 \cdot 10^6$ km

10.2 Moon parameters

Average radius:	1738 km
Mass:	$7.35 \cdot 10^{22}$ kg
Density:	$3341 \frac{kg}{m^3}$
Gravitational acceleration:	$1.6 \frac{m}{s^2}$

10.3 Sun parameters

Average radius:	696 340 km
Mass:	$1.988 \cdot 10^{30}$ kg
Density:	$1408 \frac{kg}{m^3}$
Gravitational acceleration:	$273.7 \frac{m}{s^2}$
Rotation around its own axis:	about 1 revolution in 25 days
Solar constant (mean value)	$1361 \frac{W}{m^2}$

10.4 Kepler's laws

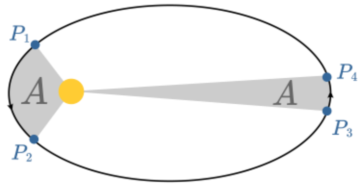
1st Kepler's law:
The planets move in elliptical orbits with the sun at one focal point.

2nd Kepler's law:

$$A_1 = A_2$$

$$A_1 = r_1 v_1 \sin(\varphi_1)$$

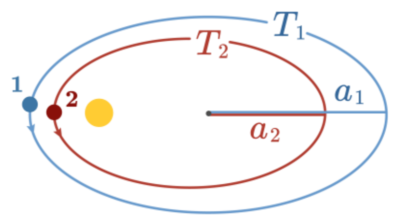
$$A_2 = r_2 v_2 \sin(\varphi_2)$$



The 2nd Kepler law states that the connecting line between the sun and the moving planet encloses equal **areas** $A = A_1 = A_2$ [m^2] in equal times. Here, A_1 [m^2] and A_2 [m^2] are two equal covered areas of the connecting line, which were covered within the same time. These areas were covered by a planet at **distance** r_1 [m] or **distance** r_2 [m] from the sun (central star) with orbital velocity v_1 [m/s] or orbital velocity v_2 [m/s]. **Angle** φ_1 [rad] and **angle** φ_2 [rad] are enclosed by the velocity direction and the connecting line.

3rd Kepler's law:

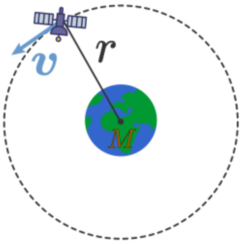
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$



Kepler's third law states that the *squares of the orbital periods* $(T_1)^2$ and $(T_2)^2$ of two planets around the central star behave like the *third powers* of the semi-major axes $(a_1)^3$ and $(a_2)^3$. To understand this, we need two planets #1 and #2. Planet #1 orbits the central star within the **orbital period** T_1 [s] and its elliptical orbit has a **semi-major axis** a_1 [m]. Planet #2 has the **orbital period** T_2 [s] around the same central star and its orbit has a large **semi-major axis** a_2 [m].

10.5 Stable orbit and escape velocity

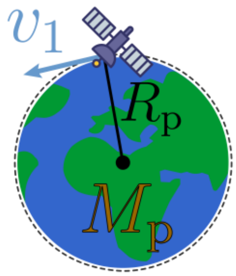
$$v = \sqrt{G\frac{M}{r}}$$



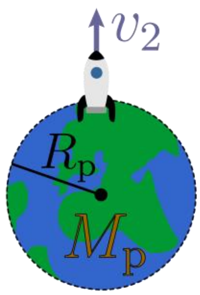
For a body (for example a satellite) to orbit without propulsion on a fixed circular path with the **radius** r [m] around a celestial body (for example earth) of the **mass** M [kg], the body must have an **orbital velocity** v [m/s]. Here G is the **gravitational constant**.

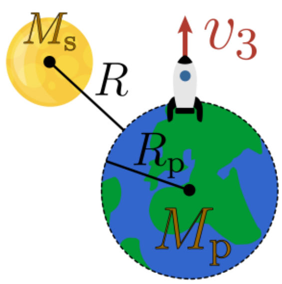
$$v_1 = \sqrt{G\frac{M_\mathrm{p}}{R_\mathrm{p}}} \qquad v_2 = \sqrt{G\frac{2M_\mathrm{p}}{R_\mathrm{p}}}$$

$$v_3 \approx \sqrt{G\left(\left(\sqrt{2}-1\right)^2\frac{M_\mathrm{s}}{R} + 2\frac{M_\mathrm{p}}{R_\mathrm{p}}\right)}$$



- The **first cosmic velocity** v_1 [m/s] is the **orbital velocity** necessary to orbit a celestial body directly at the surface on a stable orbit. Here M_p [kg] is the **mass of the planet** being orbited. And R_p [m] is the **radius of the planet**.
- The **second cosmic velocity** v_2 [m/s] is the **escape velocity** necessary to escape the gravitational attraction of a celestial body (for example, the Earth).
- The **third cosmic velocity** v_3 [m/s] is the escape velocity which is necessary to escape from the gravitational attraction of the solar system when the body starts on planet P. Here, only the gravitational field of the central star (Sun) and the planet on which the body starts its escape attempt is considered. Here M_s [kg] is the **mass of the central star** and R [m] the **average distance** of the planet to the central star.



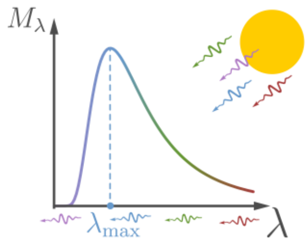


Celestial body	First cosmic velocity v_1	Second cosmic velocity v_2	Third cosmic velocity v_3
Earth	7.9 km/s	11.2 km/s	16.6 km/s
Moon	1.7 km/s	2.3 km/s	12.6 km/s

Celestial body	First cosmic velocity v_1	Second cosmic velocity v_2	Third cosmic velocity v_3
Jupiter	42 km/s	60 km/s	60.4 km/s
Sun	436 km/s	617 km/s	21.6 km/s

10.6 Wien's displacement law

$$T = \frac{2897.8 \cdot 10^{-6}\,\mathrm{mK}}{\lambda_{\mathrm{max}}}$$



Absolute temperature T [K] of a glowing body (for example sun). The glowing body radiates light of different wavelengths. The light of **wavelength** λ_{max} [m] has the *highest* intensity. Here $2897.8 \cdot 10^{-6}\,\mathrm{mK}$ is the **Wien constant**. Caution: mK is not "millikelvin", but the unit "meter times kelvin".

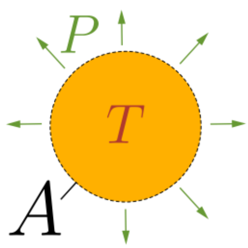
10.7 Planck's radiation law

$$L = \frac{2hc^2}{\lambda^5}\left(e^{\frac{hc}{\lambda k_{\mathrm{B}} T}} - 1\right)^{-1} \qquad L = \frac{2hf^3}{c^2}\left(e^{\frac{hf}{k_{\mathrm{B}} T}} - 1\right)^{-1}$$

Radiance L [W/m^{2}sr] of a black body as a function of its **absolute temperature** T [K] and emitted radiation of **frequency** f [Hz] and **wavelength** λ [m].

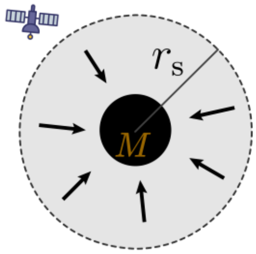
10.8 Stefan-Boltzmann law

$$P = \sigma A T^4$$



An absolute black body with **surface temperature** T [K] and **surface area** A [m^2] emits **radiant power** P [W = J/s]. Here $\sigma = 5.67 \cdot 10^{-8}$ J/m^{2}K^{4}s is the **Stefan-Boltzmann constant**.

10.9 Schwarzschild radius



$$r_s = \frac{2GM}{c^2}$$

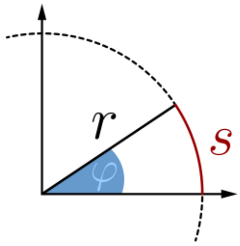
Schwarzschild radius r_s [m] is the radius of the event horizon of a black hole and indicates from which distance to a black hole of the **mass** M [kg] the light can no longer escape its gravitational attraction. Here c [m/s] is the **speed of light** and G the **gravitational constant**.

11. MATHEMATICS FOR PHYSICS

Important mathematical formulas often used in physics.

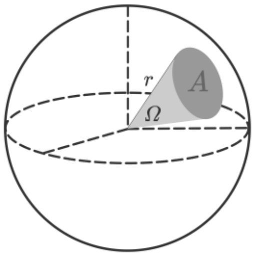
11.1 Angle (definition)

$$\varphi = \frac{s}{r} \quad \Omega = \frac{A}{r^2}$$



Angle φ [rad = 1] is defined as the ratio of the **arc length** s [m] to the **radius** r [m] of a circle. $\varphi = 1$ rad (radian) is the angle at which the arc length is equal to the radius. **Solid angle** Ω [sr = 1] is the ratio of the **partial area** A [m^2] of an (imaginary) sphere to its squared radius.

$$\varphi = \frac{x}{\pi} 180^\circ \quad x = \frac{\varphi}{180^\circ}\pi$$

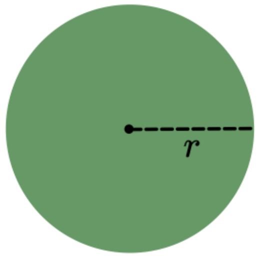


Angle φ [rad] in **degrees** can be converted to **radians** x [°] and vice versa.

11.2 Circle and ellipse

$$A = \pi r^2 \quad A = \frac{\pi}{4} d^2$$

$$U = 2\pi r$$



Area A [m^2] and **circumference** U [m] of a *circle* with **radius** r [m] and **diameter** d [m].

$$A = ({r_a}^2 - {r_i}^2)\pi$$

Area A [m^2] of a circular ring with **outer radius** r_a [m] and **inner radius** r_i [m].

$$r_1 + r_2 = 2a$$

$$e = \sqrt{a^2 - b^2} \quad \varepsilon = \frac{e}{a}$$

Linear eccentricity e [m] of an ellipse with **semi-major axis length** a [m] and **semi-minor axis length** b [m] gives the distance of a focal point to the center of the ellipse. Here r_1 [m] and r_2 [m] are connecting lines:

- r_1 is the distance of the first focal point to the point on the ellipse.
- r_2 is the distance of the second focal point.

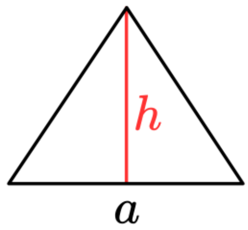
The (**numerical**) **eccentricity** ε $[-]$ is a dimensionless quantity that lies between 0 and 1 and indicates the deviation of the ellipse from a circular shape ($\varepsilon = 0$).

Planet	Eccentricity ε of the orbit
Merkur	0.2056
Venus	0.0068
Earth	0.0167
Mars	0.0934
Jupiter	0.0484
Saturn	0.0541
Uranus	0.0472
Neptun	0.0086

11.3 Triangle

$$A = \frac{1}{2} a h \quad A = \frac{1}{2} ab \sin(\gamma)$$

$$U = a + b + c \quad \alpha + \beta + \gamma = 180^\circ$$

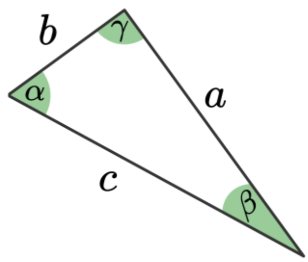


Area A $[\mathrm{m}^2]$ of a *general triangle* with **edge lengths** a, b, c [m], opposite **angles** α, β, γ [rad] and **altitude** h [m]. Here a [m] is the **base** length. The **perimeter** U [m] of the triangle is the sum of the edge lengths.

- For an *equilateral* triangle: $A = \frac{a^2}{4}\sqrt{3}$, $h = \frac{a}{2}\sqrt{3}$.
- For a *right* triangle: $A = \frac{bc}{2}$, $\sin(\alpha) = \frac{\text{Opposite cathetus}}{\text{Hypotenuse}}$, $\cos(\alpha) = \frac{\text{Adjacent cathetus}}{\text{Hypotenuse}}$

$$a = \frac{\sin(\alpha)}{\sin(\beta)} b \quad a = \frac{\sin(\alpha)}{\sin(\gamma)} c \quad b = \frac{\sin(\beta)}{\sin(\gamma)} c$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos(\gamma)}$$



The *laws of cosine and sine* are relationships between the **edge lenghts** a, b and c [m] of a general triangle and the **angles** α, β and γ [rad].

11.4 Quadrilateral (square, rectangle, parallelogram, trapezoid)

$$\alpha + \beta + \gamma + \delta = 360°$$

The sum of the interior **angles** α, β, γ and δ [rad] of a *quadrilateral* is 360 degrees.

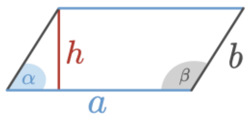
$$A = a^2 \quad U = 4a \quad d = a\sqrt{2}$$

Area A [m²], **perimeter** U [m] and **diagonal length** d [m] of a *square* with **edge length** a [m].

$$A = ab \quad U = 2a + 2b \quad d = \sqrt{a^2 + b^2}$$

Area A [m²], **perimeter** U [m] and **diagonal** d [m] of a *rectangle* with **edge lengths** a [m] and b [m].

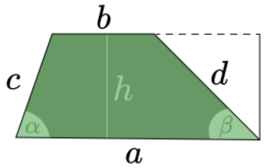
$$A = ah \quad U = 2a + 2b$$



Area A [m²] and **perimeter** U [m] of a *parallelogram* with **altitude** h [m] and **edge lengths** a [m] and b [m] of the base.

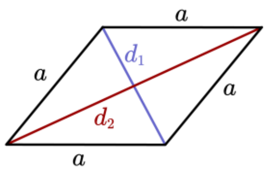
$$A = \frac{1}{2}(a + b)h$$

$$h = c\sin(\alpha)$$



Area A [m²] of a *trapezoid* depends on the **edge lengths** a [m] and b [m] and on the **altitude** h [m]. The altitude can be calculated with the **edge length** c [m] and the **angle** α [rad].

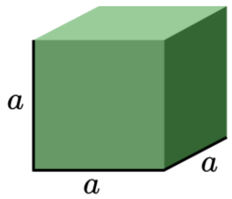
$$A = \frac{1}{2}d_1 d_2$$



Area A [m²] of a *rhombus* depends on its **diagonal lengths** d_1 [m] and d_2 [m].

11.5 Cube

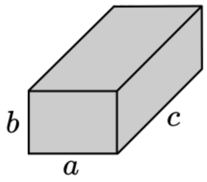
$$A = 6a^2 \qquad V = a^3$$



Area A [m²] and **volume** V [m³] of a *cube* depend on its **edge length** a [m].

11.6 Cuboid

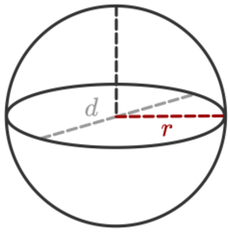
$$A = 2(ab + ac + bc) \qquad V = abc$$



Area A [m²] of a cuboid depends on its **edge lengths** a, b and c [m].

11.7 Sphere

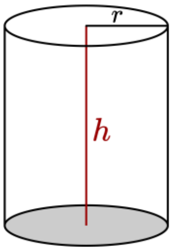
$$A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3 \quad V = \frac{\pi}{6}d^3$$



Volume V [m³] of a *sphere* depends on its **radius** r [m] or its **diameter** d [m].

11.8 Cylinder

$$A = 2\pi r(r + h) \qquad V = \pi r^2 h$$



Area A [m²] and **volume** V [m³] of a *right circular cylinder* of **height** h [m] and whose base has **radius** r [m].

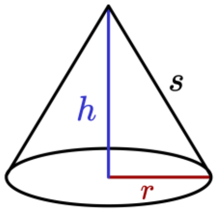
$$V = (r_a{}^2 - r_i{}^2)\pi h$$

Volume V [m³] of a *hollow cylinder* with **outer radius** r_a [m] and **inner radius** r_i [m].

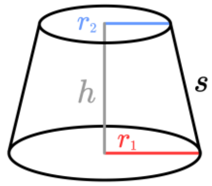
11.9 Cone

$$V = \frac{\pi}{3}hr^2 \qquad A = \pi r(r + s) \quad A_m = \pi r s$$

$$s = \sqrt{h^2 + r^2}$$



Volume V [m^3], **surface area** A [m^2] and **lateral area** A_m [m^2] of a *circular cone* of **height** h [m], **radius** r [m] and **side length** s [m].



$$V = \frac{\pi h}{3}\left(r_1{}^2 + r_2{}^2 + r_1 r_2\right) \qquad s = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$A_\mathrm{m} = \pi s(r_1 + r_2) \qquad A = \pi\left(r_1{}^2 + r_2{}^2 + s(r_1 + r_2)\right)$$

Volume V [m^3], **surface area** A [m^2] and **lateral area** A_m [m^2] of a *right circular frustum* of **height** h [m], **radius** r_1 [m] of base, **radius** r_2 [m] of top area and **side length** s [m].

11.10 Sine, cosine and tangens

$$\sin(90^\circ - \alpha) = \cos(\alpha) \qquad \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

$$\sin(-\alpha) = -\sin(\alpha) \qquad \tan(-\alpha) = \tan(\alpha)$$

$$\cos(-\alpha) = \cos(\alpha) \qquad \sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \qquad \cos(2\alpha) = 2\cos(\alpha)^2 - 1$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

11.11 Solving a quadratic equation (pq formula)

$$x_1 = -\frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q} \qquad x_2 = -\frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

The **quadratic equation** $x^2 + px + q = 0$ has two solutions x_1 and x_2.

11.12 Power and root laws

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^n b^n = (ab)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$$

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$$

11.13 Logarithm laws

$$b^c = a \leftrightarrow c = \log_b(a)$$

$$\mathrm{e}^{\ln(x)} = x$$

$$\log_c(a) + \log_c(b) = \log_c(ab)$$

$$\ln(\mathrm{e}^x) = x$$

$$\log_c(a) - \log_c(b) = \log_c\left(\frac{a}{b}\right)$$

$$\log_c(a^n) = n\log_c(a)$$

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$$

$$\log_a(a^n) = n$$

$$\log_c(\sqrt[n]{a}) = \frac{1}{n}\log_c(a)$$

11.14 Series

Geometric sequence: $$1 + q + q^2 + q^3 + \cdots + q^n = \frac{1-q^n}{1-q}$$

Arithmetic sequence: $$a_1 + a_2 + a_3 + \cdots + a_n = \frac{n(a_1 + a_n)}{2}$$

Sum of natural numbers: $$1 + 2 + 3 + \cdots + n = \frac{n}{2}(n+1)$$

Sum of even numbers: $$2 + 4 + 8 + \cdots + 2n = n(n+1)$$

Sum of odd numbers: $1+3+5+\cdots+(2n-1) = n^2$

Sum of the square numbers: $1+4+9+\cdots+n^2 = \frac{n}{6}(n+1)(2n+1)$

Sum of the cubic numbers: $1+8+27+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$

11.15 Binomial formulas

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

11.16 Statistics

$$\mu = x_1p_1 + x_2p_2 + x_3p_3 + \cdots \qquad \overline{x} = \frac{x_1 + x_2 + x_3 + \ldots}{N}$$

Expected value μ of a random variable X with the **measured values** $x_1, x_2, x_3, \ldots$ and associated **probabilities** $p_1, p_2, p_3, \ldots$ to get these values out. **Empirical mean** $\overline{x}$ is the sum of the measured values $x_1, x_2, x_3, \ldots$ divided by the **number of measured values** N. The expected value is the theoretically expected mean value.

$$\sigma^2 = (x_1-\mu)^2p_1 + (x_2-\mu)^2p_2 + (x_3-\mu)^2p_3 + \ldots$$

$$\sigma_{\mathrm{e}}^2 = \frac{(x_1-\overline{x})^2 + (x_2-\overline{x})^2 + (x_3-\overline{x})^2 + \ldots}{N-1}$$

Variance σ^2 gives the sum of squared deviations $(x_1-\mu)^2$, $(x_2-\mu)^2$ and so on from the **expected value** μ. Here σ_{e}^2 is the **empirical variance** and N is the **number of measured values**.

$$\sigma = \sqrt{(x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + (x_3 - \mu)^2 p_3 + \ldots}$$

$$\sigma_e = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + (x_3 - \overline{x})^2 + \ldots}{N - 1}}$$

(**Empirical**) **standard deviation** σ or σ_e indicates how much the **measured values** $x_1, x_2, x_3, \ldots$ deviate on average from the **expected value** μ. Here N is the **number of measured values**. If $\overline{x}$ is the mean and the measured values are *normally distributed*, then:

- In the range $\overline{x} \pm \sigma$ lie **68%** of all measured values.
- In the range $\overline{x} \pm 2\sigma$ lie **95.4%** of all measured values.
- In the range $\overline{x} \pm 3\sigma$ lie **99.7%** of all measured values.

$$\sigma(\overline{x}) = \frac{\sigma_e}{\sqrt{N}}$$

The **standard deviation** $\sigma(\overline{x})$ of the **mean** $\overline{x}$ with N **measured values**. The doubling of the accuracy needs a *quadrupling* of the number of measured values!

- When *multiplying* $\overline{x}_1 \cdot \overline{x}_2$ and *dividing* $\overline{x}_1 / \overline{x}_2$ of two means, their **relative errors** f_1 and f_2 add up to a **total relative error**: $f = f_1 + f_2$.
- When *adding* $\overline{x}_1 + \overline{x}_2$ and *subtracting* $\overline{x}_1 - \overline{x}_2$ of two mean values, their **absolute errors** Δx_1 and Δx_2 add up to a **total absolute error**: $\Delta x = \Delta x_1 + \Delta x_2$.

$$\binom{n}{k} = \frac{n!}{k!\,(n - k)!}$$

Binomial coefficient "n over k" indicates *how many possibilities* there are to select k objects each from n different objects.

KEYWORDS OF PHYSICS

Find out on which pages a physical quantity occurs.